A receding horizon approach to short term electricity markets

Joe Warrington, Paul Goulart,
Sébastien Mariéthoz, Manfred Morari

HYCON2 Workshop on Energy, Brussels
3rd September 2012

Automatic Control Laboratory, ETH Zurich

www.control.ee.ethz.ch

1. Motivation

Power system with

- High wind power penetration (>> 20% of peak load by capacity)
- Poor predictions of wind power availability (e.g. timing of weather fronts)
- Significant storage (e.g. hydroelectric, flexible consumers)

Obvious:

Static daily patterns can no longer be used to plan thermal generator operation (unit commitment and dispatch).

Not as obvious:

Even if efficient generation plan can be found centrally, can't necessarily be achieved using existing market mechanisms.

1. Motivation

This is hard!

Consider a multi-period auction where participants don't know what prices to expect:

- Bidding strategies are a function of time-coupled expected prices
- But the resulting prices are determined by those bids

One-shot discovery of efficient prices is impossible here.

Market mechanism design goals:

- Efficient incorporation of wind power (over relevant timescale)
- Correct time-coupled price incentives for participants
- Timely use of new forecasts
- Make reasonable demands of participants

Contents

2. Multi-period optimal power flow

Minimize load satisfaction costs over a time horizon, subject to participant and network constraints.

1. AC network model:

$$
\begin{aligned}
\min_{\{p_i\},\{q_i\},\{v_k\}} \; & \sum_{i \in \mathcal{C}} J_i(p_i, q_i) & \\
\text{s. t. } & (p_i, q_i) \in \mathcal{PQ}_i & \forall i \in \mathcal{C}, \\
& \sum_{i \in \mathcal{C}_n} [p_i + \hat{p}_i]_k = \mathrm{Re}\{v_k^* Y_n v_k\} & \forall n \in \mathcal{N}, \forall k, \\
& \sum_{i \in \mathcal{C}_n} [q_i + \hat{q}_i]_k = \mathrm{Re}\{v_k^* \overline{Y}_n v_k\} & \forall n \in \mathcal{N}, \forall k, \\
& \underline{v}_n \leq |[v_k]_n| \leq \overline{v}_n & \forall n \in \mathcal{N}, \forall k, \\
& |v_k^* Y_{lm} v_k| \leq \overline{S}_{lm} & \forall (l, m) \in \mathcal{L}, \forall k
\end{aligned}
$$

Both real power *p* and reactive power *q* considered.

Bus angles **nonlinear** in nodal power injections.

Complex voltage phasors are **magnitude-constrained**.

Line power flow limits are **2nd-order cone constraints**.

2. Multi-period optimal power flow

Minimize load satisfaction costs over a time horizon, subject to participant and network constraints.

1. AC network model:

$$
\begin{aligned}
&\min_{\{p_i\},\{q_i\},\{W_k\}} \sum_{i\in\mathcal{C}} J_i(p_i,q_i) \\
\text{s. t. } & (p_i,q_i)\in\mathcal{PQ}_i && \forall i\in\mathcal{C} \\
& \langle \mathbf{Y}_n, W_k\rangle - \sum_{i\in\mathcal{C}_n}[p_i+\hat{p}_i]_k = 0 && \forall n\in\mathcal{N},\ \forall k, \\
& \langle \overline{\mathbf{Y}}_n, W_k\rangle - \sum_{i\in\mathcal{C}_n}[q_i+\hat{q}_i]_k = 0 && \forall n\in\mathcal{N},\ \forall k, \\
& \underline{v}_n^2 \le \langle M_n, W_k\rangle \le \overline{v}_n^2 && \forall n\in\mathcal{N},\ \forall k, \\
& \langle \mathbf{Y}_{lm}, W_k\rangle^2 + \langle \overline{\mathbf{Y}}_{lm}, W_k\rangle^2 \le \overline{S}_{lm}^2 && \forall (l,m)\in\mathcal{L},\ \forall k, \\
& W_k \succeq 0 && \forall k, \\
& \boxed{\operatorname{rank}(W_k) = 1 \qquad \forall k}
\end{aligned}
$$

Semidefinite relaxation achieved using a **change of variables**:

$$
\boxed{W_k := \begin{bmatrix} \mathrm{Re}\{v_k\} \\ \mathrm{Im}\{v_k\} \end{bmatrix} [\mathrm{Re}\{v_k\}' \ \mathrm{Im}\{v_k\}']}
$$

Must obtain a **rank-1 solution** to extract usable solution *v*.

Relaxation turns out to be exact: rank-1 optimal solution under mild assumptions even without enforcing rank!

2. Multi-period optimal power flow

Minimize load satisfaction costs over a time horizon, subject to participant and network constraints.

2. DC-linearized network model:

$$
\begin{aligned}
\min_{\{\{p^{\mathrm{e}}_{im^{\mathrm{e}}}\}_{m^{\mathrm{e}}=1}^{n_i^{\mathrm{e}}}\}_{i=1}^{N_{\text{nodes}}}} \quad & \sum_{i=1}^{N_{\text{nodes}}} \left[\sum_{m^{\mathrm{e}}=1}^{n_i^{\mathrm{e}}} J_{im^{\mathrm{e}}}(p^{\mathrm{e}}_{im^{\mathrm{e}}}) \right] \\
\text{s.t.} \quad & \sum_{i=1}^{N_{\text{nodes}}} \left[\sum_{m^{\mathrm{e}}=1}^{n_i^{\mathrm{e}}} p^{\mathrm{e}}_{im^{\mathrm{e}}} + \sum_{m^{\mathrm{i}}=1}^{n_i^{\mathrm{i}}} \hat{p}^{\mathrm{i}}_{im^{\mathrm{i}}} \right] = 0, \\
& \sum_{i=1}^{N_{\text{nodes}}} A_i \left[\sum_{m^{\mathrm{e}}=1}^{n_i^{\mathrm{e}}} p^{\mathrm{e}}_{im^{\mathrm{e}}} + \sum_{m^{\mathrm{i}}=1}^{n_i^{\mathrm{i}}} \hat{p}^{\mathrm{i}}_{im^{\mathrm{i}}} \right] \leq \overline{P}, \\
& p^{\mathrm{e}}_{im^{\mathrm{e}}} \in \mathcal{P}^{\mathrm{e}}_{im^{\mathrm{e}}} \cap \mathcal{S}^{\mathrm{e}}_{im^{\mathrm{e}}} \;\; \forall i, m^{\mathrm{e}}=1,\ldots,n_i^{\mathrm{e}}
\end{aligned}
$$

Only real power p considered.

Bus angles **linear** in nodal power injections.

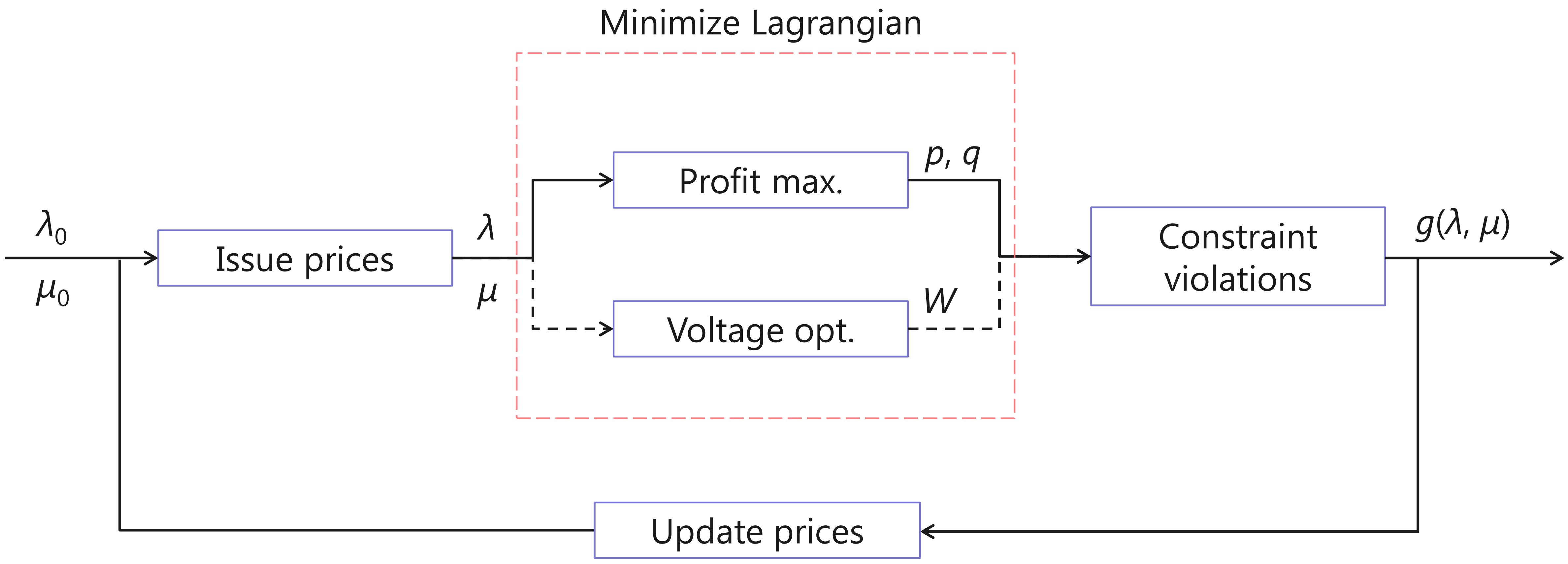
Voltage magnitudes constant, no power losses.

Line power flow constraints linear in power injections.

2. Solution mechanism

Lagrangian Relaxation algorithm:

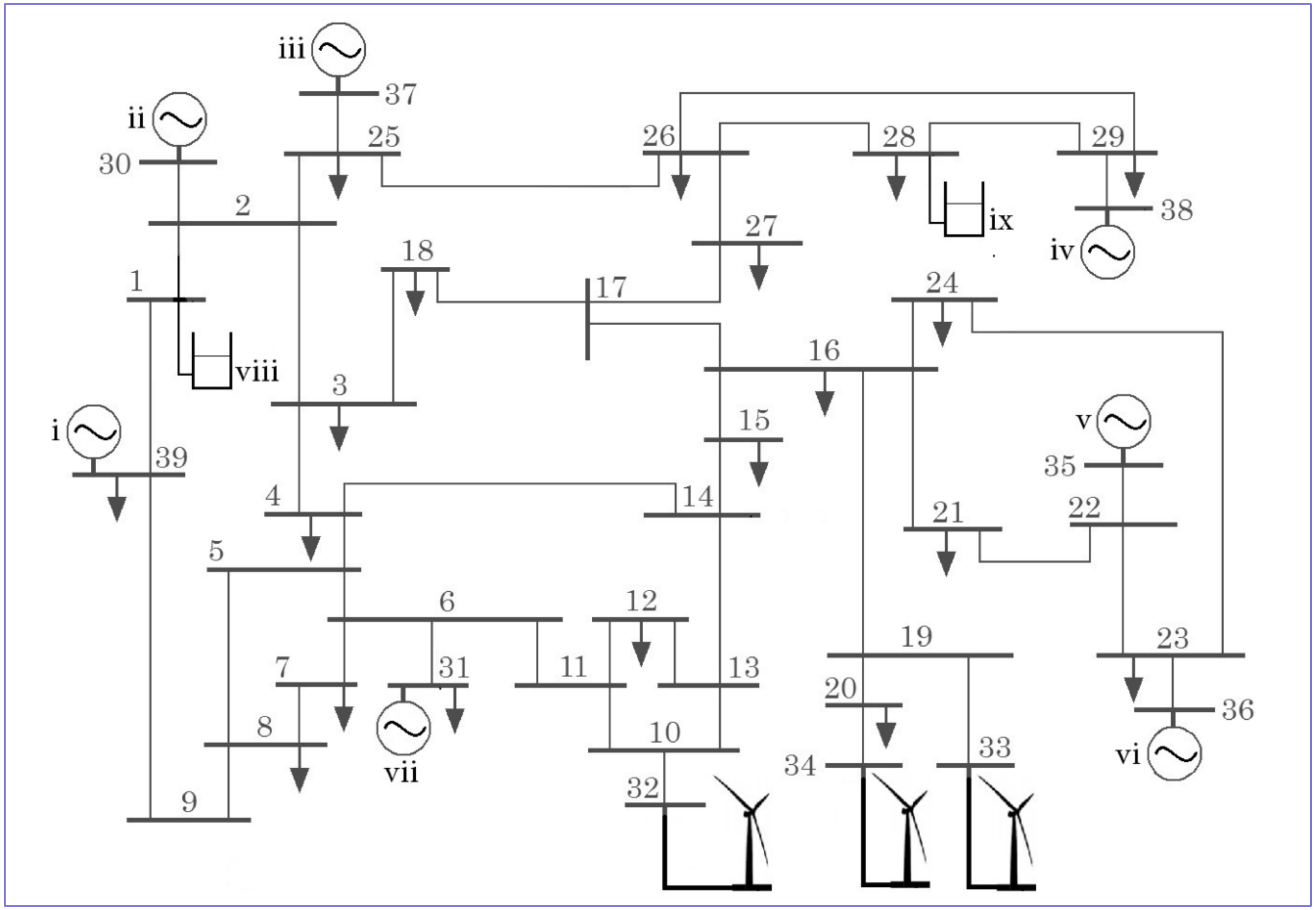
$$L(p,q,W,\lambda,\mu) := \sum_{n\in\mathcal{N}} \sum_{i\in\mathcal{C}_n} \left(J_i(p_i,q_i) - \lambda_n' p_i - \mu_n' q_i\right) + \sum_{k=1}^{T} \left\langle \sum_{n\in\mathcal{N}} \left(\lambda_{nk}\mathbf{Y}_n + \mu_{nk}\overline{\mathbf{Y}}_n\right), W_k \right\rangle$$



- Feasible and optimal solution obtained, under convexity assumptions.
- Also constitutes an iterative market mechanism.

2. Case studies

Carried out on the IEEE 39 bus New England network



Network:

39 buses with ±5% voltage limits

Nom. voltage 230 kV

46 lines with line flow limits

Participants:

7 generators: ramp limits, *p*, *q* constraints

2 storage units

3 wind farms

19 inelastic loads

Fixed load and wind power schedules

2. Case studies: AC network model

Time horizon length T = 10: Each iteration involves solving 9 QPs (for the price-elastic participants) and 10 SDPs (for the network voltages).

Method	Iterations	Infeasibility	Cost
Basic subgradient	Thousands….	↘ 0	7.0292×10^6
Aggregation (1.) + basic subgradient	~600	~5	7.0292×10^6
Aggregation (1.) + prioritizing feasibility (2.)	144	10^{-6}	7.1200×10^6 (+1.3%)

Further iteration reductions from heuristics clearly possible.

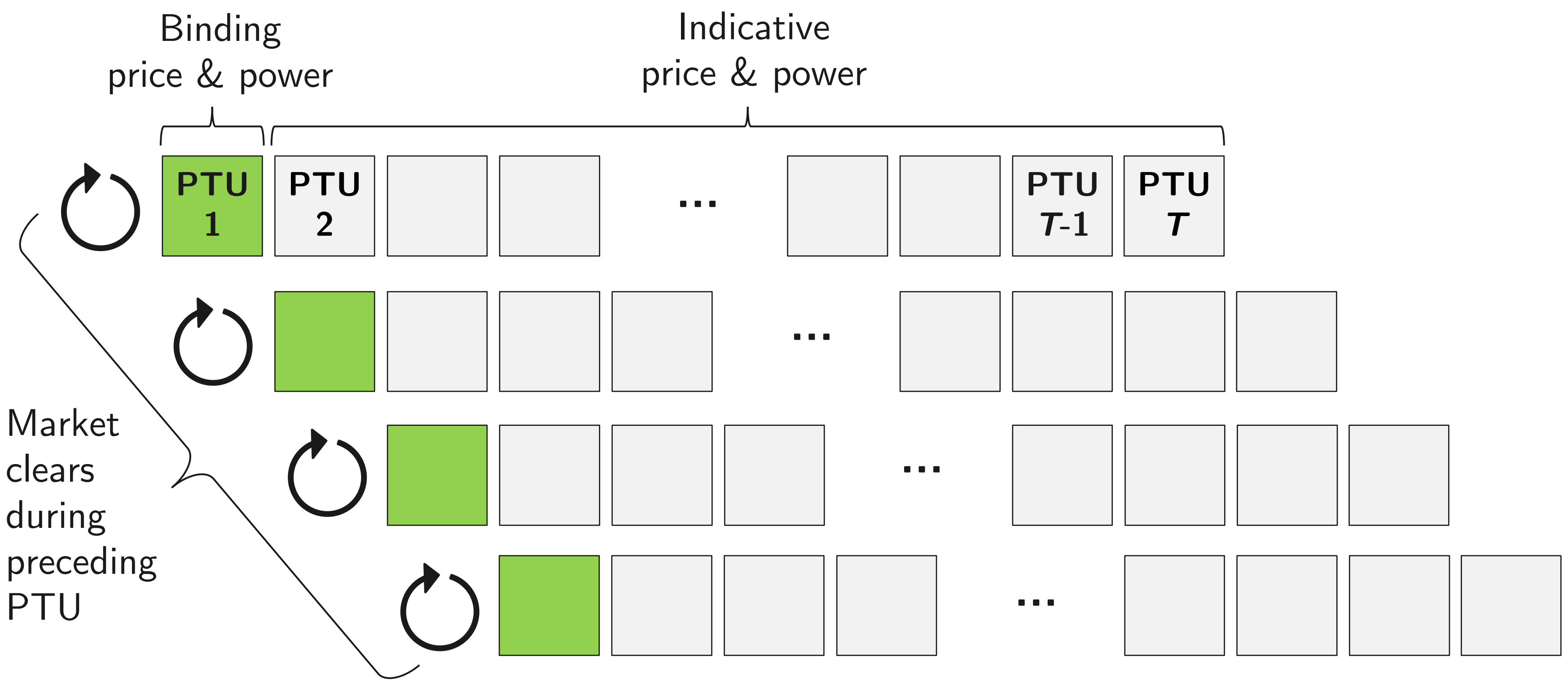
J. Warrington, P. J. Goulart, S. Mariéthoz, and M. Morari, "**A market mechanism for solving multi-period optimal power flow exactly on AC networks with mixed participants**," *American Control Conference*, Montreal, Canada, 2012.

2. Case studies: DC network model

Receding horizon market clearing:

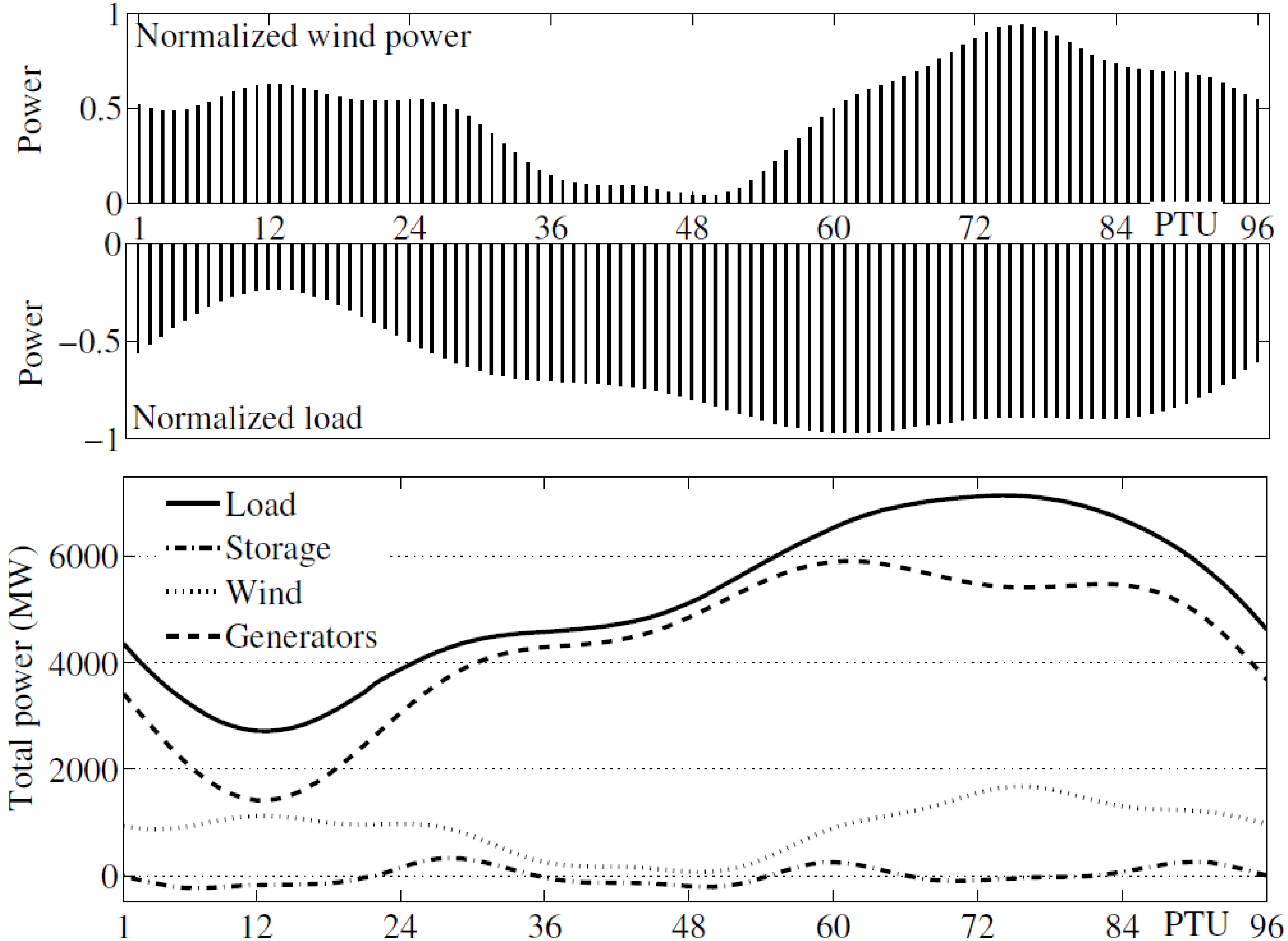
Already exists to some extent (e.g. Spain), but not formalized or analyzed carefully yet.

Constantly-updating price forecast ensures best use of new information.

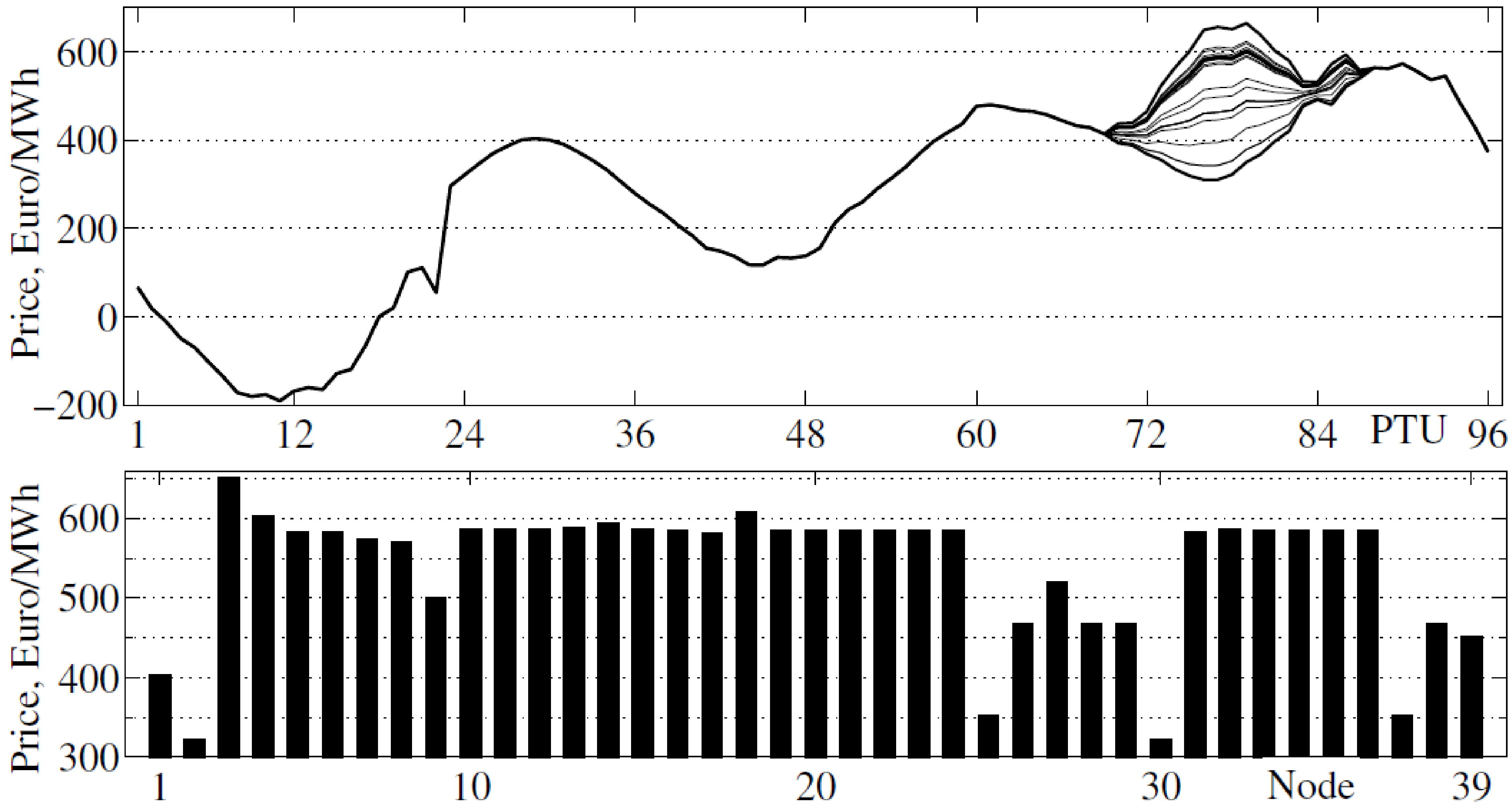


2. Case studies: DC network model

Time horizon length $T = 24$. Wind is predicted with an expected error magnitude that grows linearly along the horizon.



2. Case studies: DC network model



Cost comparison:

	Method	Cost of dispatch	Cost vs. method *1)*
1)	Global prescient	1.0632×10^8	-
2)	Global rec. hor.	1.0822×10^8	+1.7%
3)	Neg. Pred. Disp.	1.0853×10^8	+2.1%
4)	Isolated PTUs	1.1185×10^8	+5.2%

J. Warrington, S. Mariéthoz, and M. Morari, "**Negotiated predictive dispatch: Receding horizon nodal electricity pricing for wind integration**," in *European Energy Market, Int. Conf. on the*, Zagreb, Croatia, 2011.

3. Affine policies for robust wind integration

Standard approach to operating network reserves:

- Assemble nominal power schedules (E-programs) from market outcome
- Size reserves according to reliability criteria, on a slower timescale, e.g. months ahead of delivery time.

Costs of reserve provision are ballooning in high wind-power markets!

A possible solution: Couple reserve provision duties with spot market trading outcome (in a sensible way)...

Trade in functional relationships between forecast error and changes in operating point of market participants.

Reduce costs by choosing schedules that are not just fixed but able to change in future, when more information will become available! These are *policies*.

3. Affine policy control problem

Power system entities (generators, storage units, loads, renewable infeeds, smart appliance aggregations…) are modelled with linear dynamics:

$$\mathbf{x}^i = A_i x_0^i + B_i \mathbf{u}^i,$$

where

$$A_i := \begin{bmatrix} \tilde{A}_i \\ \tilde{A}_i^2 \\ \vdots \\ \tilde{A}_i^T \end{bmatrix}, \quad B_i := \begin{bmatrix} \tilde{B}_i & 0 & \cdots & 0 \\ \tilde{A}_i \tilde{B}_i & \tilde{B}_i & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \tilde{A}_i^{T-1} \tilde{B}_i & \cdots & \tilde{A}_i \tilde{B}_i & \tilde{B}_i \end{bmatrix}.$$

and/or an associated uncontrollable reference and a prediction error

$$r_i + G_i \delta \qquad \{\delta : S\delta \le h\}$$

Costs assumed to be *quadratic* in state and input (with matrices PSD):

$$J_i(\mathbf{x}^i, \mathbf{u}^i) := f_i^{x\prime} \mathbf{x}^i + \frac{1}{2} \mathbf{x}^{i\prime} H_i^x \mathbf{x}^i + f_i^{u\prime} \mathbf{u}^i + \frac{1}{2} \mathbf{u}^{i\prime} H_i^u \mathbf{u}^i$$

Constraints polyhedral:

$$\mathcal{Z}_i := \left\{ \begin{bmatrix} \mathbf{x}^i \\ \mathbf{u}^i \end{bmatrix} : [T_i \; U_i] \begin{bmatrix} \mathbf{x}^i \\ \mathbf{u}^i \end{bmatrix} \le v_i \right\}$$

3. Affine policy control problem

Would like a sequence $\mathbf{u}^i = \pi_i(\delta)$ for each *i* that guarantees production = consumption and line constraints satisfied for any sequence $\delta \in \Delta$.

Problem intractable for general causal policies π_i – restrict to affine policies for each agent *i* :

$$\mathbf{u}^i = D_i \delta + e_i$$

Lower triangular structure required for causality:

$$D_i = \begin{bmatrix} [D_i]_{1,1} & 0 & \cdots & 0 \\ [D_i]_{2,1} & [D_i]_{2,2} & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ [D_i]_{T,1} & \cdots & [D_i]_{T,T-1} & [D_i]_{T,T} \end{bmatrix}$$

At time when policy is chosen, assume prediction error δ enters as finite-dimensional disturbance from a *bounded* set Δ, with *known mean and covariance properties.*

3. Affine policy control problem

$$\min_{(D_i,e_i)\in\mathcal{F}_i(x_0^i)} \quad \sum_{i=1}^{N_p} \tilde{J}_i(x_0^i, D_i, e_i)$$

$$\text{s.t.} \sum_{i=1}^{N_p} r_i + G_i\delta + C_i(A_i x_0^i + B_i(D_i\delta + e_i)) = 0\,, \quad \forall \delta \in \Delta\,,$$

$$\sum_{i=1}^{N_p} \Gamma_i(r_i + G_i\delta + C_i(A_i x_0^i + B_i(D_i\delta + e_i))) \leq \overline{p}\,, \quad \forall \delta \in \Delta\,.$$

Constraints:

- Match supply with demand
- Transmission line current constraints (e.g. $N - 1$ security)
- Local constraints (ramping, storage capacity limits, power output bounds)

Challenge:

Choose an optimal causal affine policy that retains feasibility for all realizations of the disturbance: not a finite constraint!

3. Affine policy control problem

"LP trick" for reformulating semi-infinite constraints (Guslitser, Ben-Tal ~2004)

$$\sum_{i=1}^{N_p} \Gamma_i(r_i + G_i\delta + C_i(A_i x_0^i + B_i(D_i\delta + e_i))) \leq \overline{p},\ \forall \delta \in \Delta$$

$$\{\delta \ : \ S\delta \ \leq \ h\}$$

$$\Updownarrow$$

$$\max_{\delta\in\Delta} \sum_{i=1}^{N_p} \Gamma_i(G_i + C_i B_i D_i)\delta + \sum_{i=1}^{N_p} \Gamma_i C_i B_i e_i$$

$$\leq \overline{p} - \sum_{i=1}^{N_p} \Gamma_i(r_i + C_i A_i x_0^i)$$

$$\Updownarrow$$

$$\left\{ \begin{array}{l} \exists Z : Z'h + \sum_{i=1}^{N_p} \Gamma_i C_i B_i e_i \leq \overline{p} - \sum_{i=1}^{N_p} \Gamma_i(r_i + C_i A_i x_0^i), \\ \sum_{i=1}^{N_p} \Gamma_i(G_i + C_i B_i D_i) = Z'S, \text{ and } Z \geq 0 \text{ element-wise.} \end{array} \right\}$$

New optimization variable *Z* introduced. Resulting formulation is in an equivalent convex and tractable form.

3. Pricing affine policies

Introduce Lagrange multipliers for market clearing and line limits and form partial Lagrangian:

$$\begin{aligned} L(Z, D_1, e_1, \ldots, D_{N_p}, e_{N_p}, \lambda, \Pi, \nu, \Psi) = \\ \sum_{i=1}^{N_p} (\tilde{J}_i(x_0^i, D_i, e_i) - \lambda_i' e_i - \langle \Pi_i, D_i \rangle) \\ + \nu' Z' h - \langle \Psi, Z' S \rangle \\ + \text{constant terms w.r.t. primal variables} \end{aligned}$$

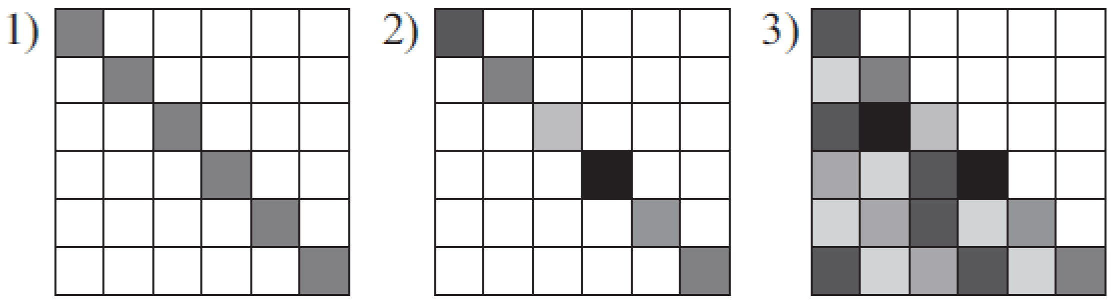
Minimizing Lagrangian corresponds to local profit maximization, and strong duality holds → efficient market prices can be derived for affine policies.

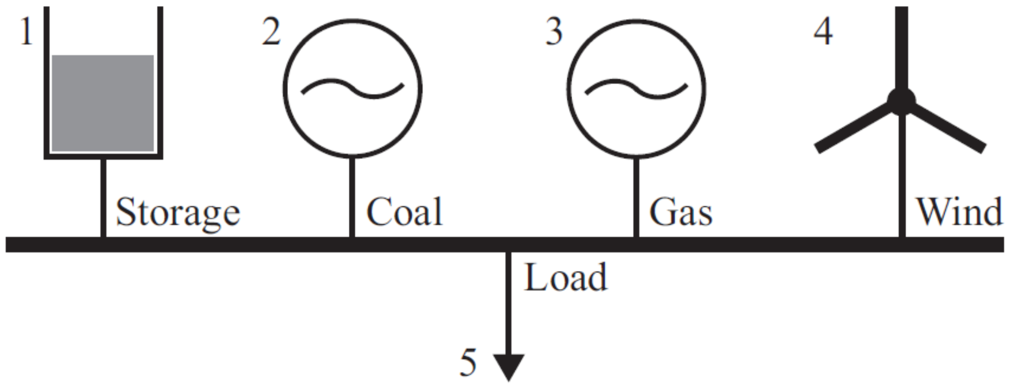
Conventional nodal power price: $\lambda_i \; := \; -B_i' C_i' (\lambda + \Gamma_i' \nu)$

Matrix of reserve policy prices: $\Pi_i \; := \; -B_i' C_i' (\Pi + \Gamma_i' \Psi)$

3. Illustrative example

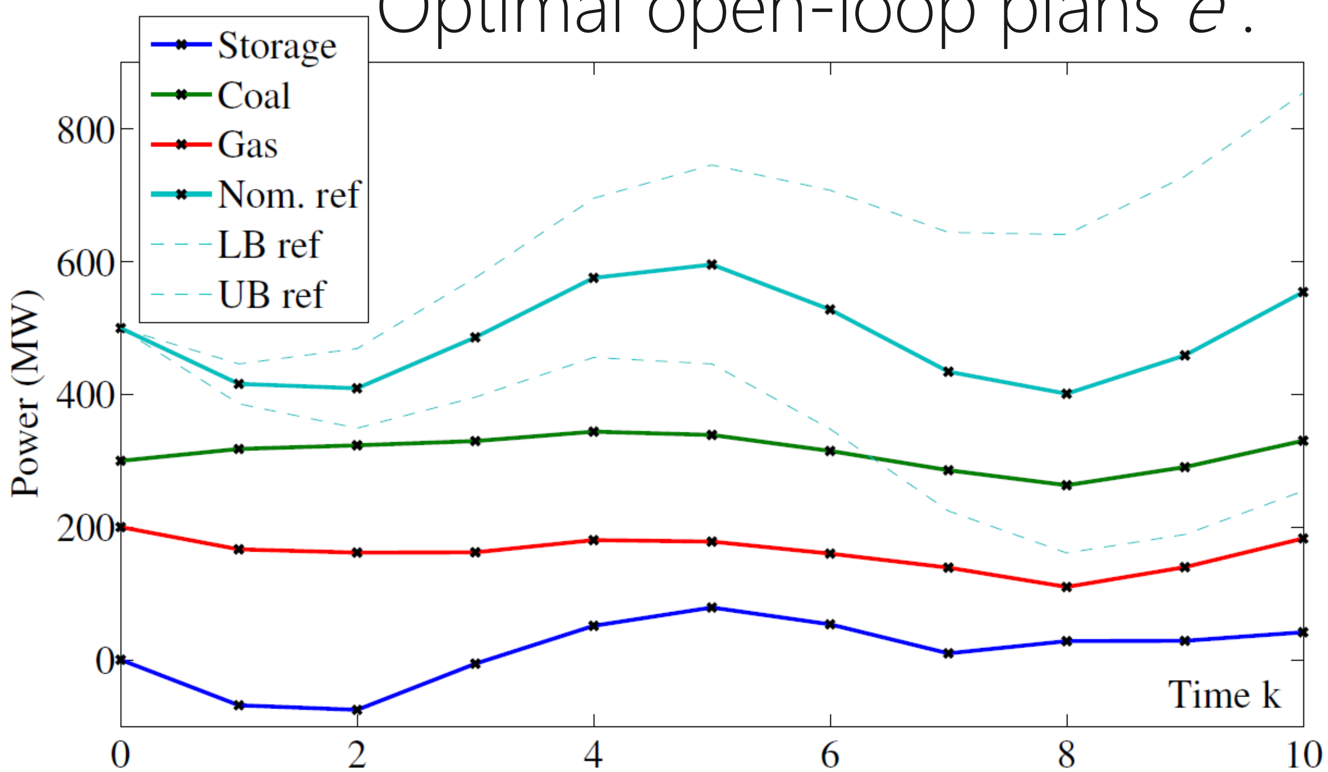
Comparing 3 different reserve schemes by restricting D structure:



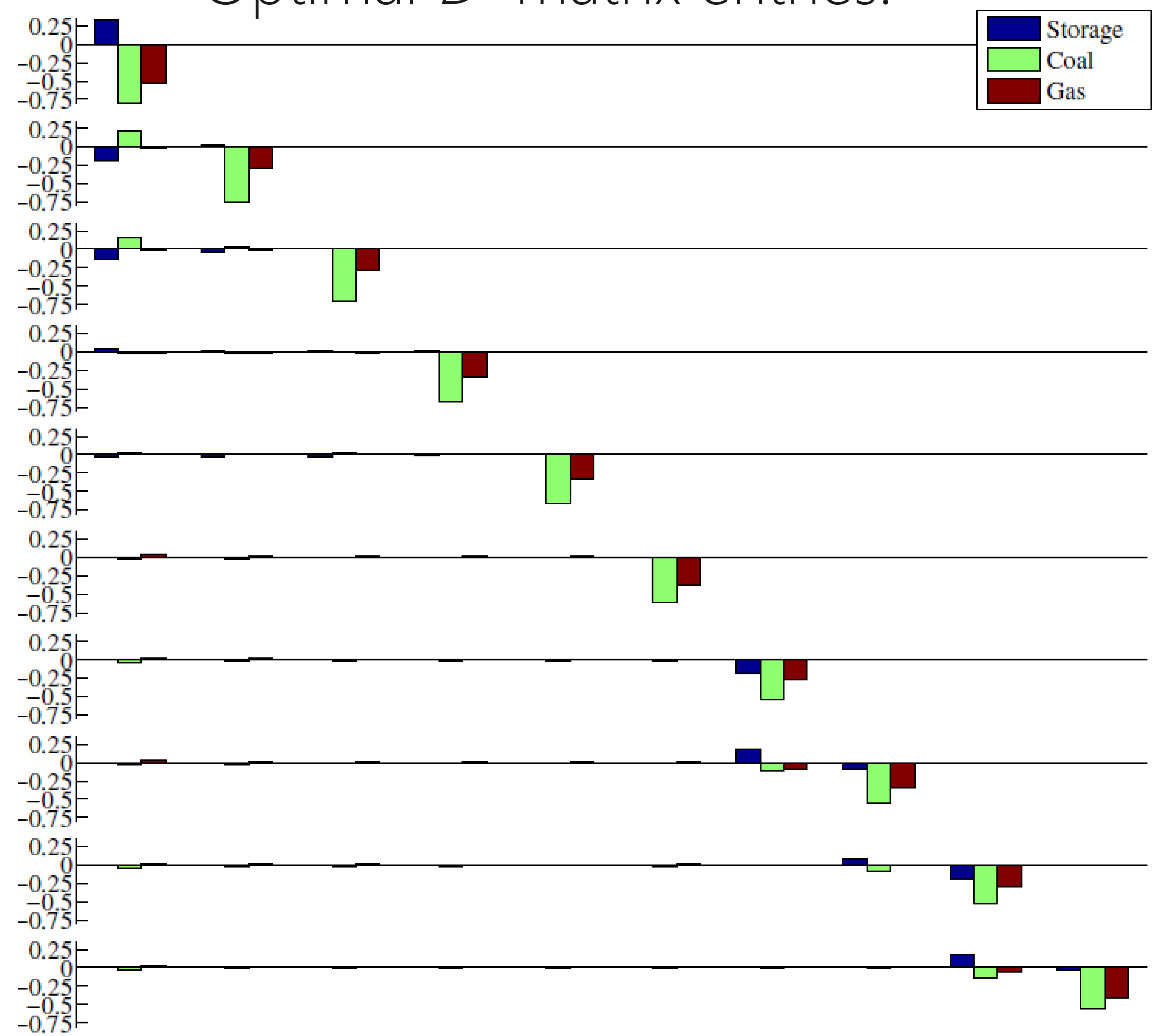


Sch.	Yalmip	CPLEX	Total cost $\times 10^5$	Reserve cost $\times 10^5$
1	566 ms	265 ms	1.9945	0.1059
2	562 ms	233 ms	1.9524 (-2.11 %)	0.0638 (-39.8 %)
3	560 ms	446 ms	1.9054 (-4.46 %)	0.0168 (-84.1 %)

Optimal open-loop plans e:



Optimal D matrix entries:



4. Conclusions

- Multi-period optimal power flow for both full AC and "DC-approximated" networks can be cast as convex problems:
  - Globally optimal market clearing available under convexity assumptions on the participants.
  - Computational complexity is much lower for the DC simplification, allowing further studies into **receding horizon prices**.

- Dual subgradient methods are compatible with privacy requirements of markets. They allow discovery of efficient prices, and constitute a **market mechanism**.

- **Cost savings** are shown for a New England grid reduction compared to a conceptual model of existing mechanisms.

- Current work involves finding a trade-off between the attractive properties of **auctions** and the advantages of Lagrangian Relaxation.

4. Conclusions

- Introduced the concept of **reserves based on *affine policies*** to reduce the cost of accommodating bounded uncertainties in short-term power system operation.

- Finite horizon optimization is semi-infinitely constrained; a tractable reformulation was derived using consideration of dual variables.

- **Efficient market prices** exist for reserve policies and mimic conventional LMPs.

- Current work: demonstrating closed-loop (receding horizon) benefits, and sale of affine policies as reserve products.

Thanks

E-Price (EU FP7 project) www.e-price-project.eu

Partners: TU Eindhoven (NL), IMT Lucca (IT), Uni. Zagreb (HR), ETH Zurich (CH), ABB (CH), TenneT (NL), APX (NL), ORS (IT), KEMA (NL)

References

J. Warrington, P. J. Goulart, S. Mariéthoz, and M. Morari, "**Robust reserve operation in power systems using affine policies**," *Conference on Decision and Control*, Maui, Hawaii, USA, 2012.

J. Warrington, P. J. Goulart, S. Mariéthoz, and M. Morari, "**A market mechanism for solving multi-period optimal power flow exactly on AC networks with mixed participants**," *American Control Conference*, Montreal, Canada, 2012.

J. Warrington, S. Mariéthoz, and M. Morari, "**Negotiated predictive dispatch: Receding horizon nodal electricity pricing for wind integration**," in *European Energy Market, Int. Conf. on the*, Zagreb, Croatia, 2011.