# Coordinated Control of distributed energy resources in the power distribution grid:
# The reactive power compensation

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# Outline

- **Introduction to distributed (leaderless) decision models**
  - Scientific context
  - Application examples
- **Application to the control of the electric power distribution networks**
  - Centralized vs decentralized decision models for the microgrid control
  - The reacitve power compensation
  - A model the microgrid
  - Minimization of the power losses by reactive power compensation
  - A quadratic approximation
  - A distributed algorithm for the reactive power compensation
  - Convergence of the algorithm

# Distributed (leaderless) decision models

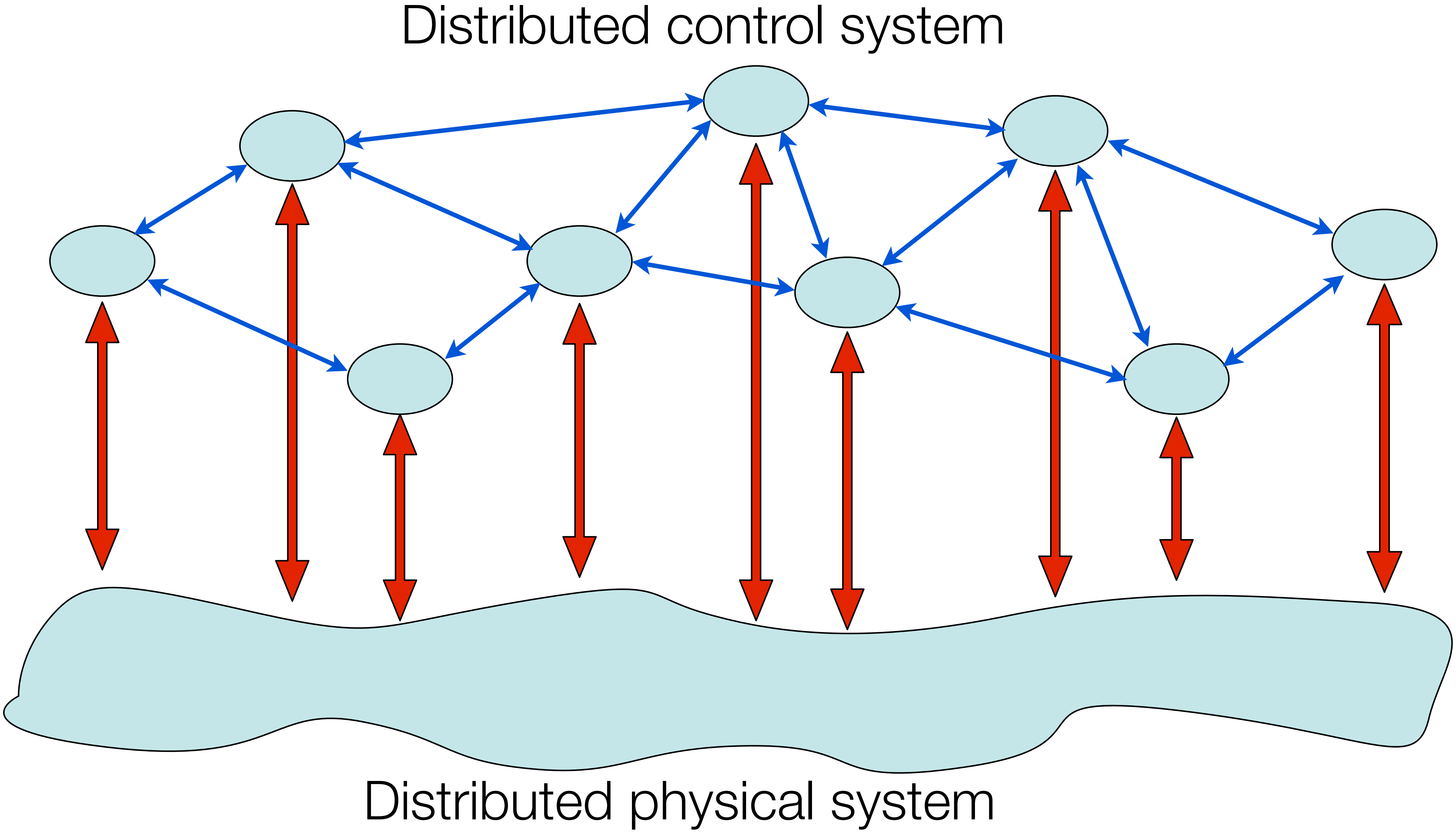
In the context of optimization, control, estimation, decision making, computation, etc, the word **DISTRIBUTED** is used with different meanings:

- The task is distributed over many agents in order to speed up the task completion (i.e. parrallel computers).
- The system itself is constituted by several interacting parts which need to be coordinated (i.e. wireless sensor networks).

In the context of the distributed decision models we can distinguish:

- Distributed decision models with leaders or with a hierarchy (based on spanning trees construction).
- Leaderless distributed decision models in which the agents are peers in the network. Here the goal is not perfomance, but the robustness and the of self-organization.

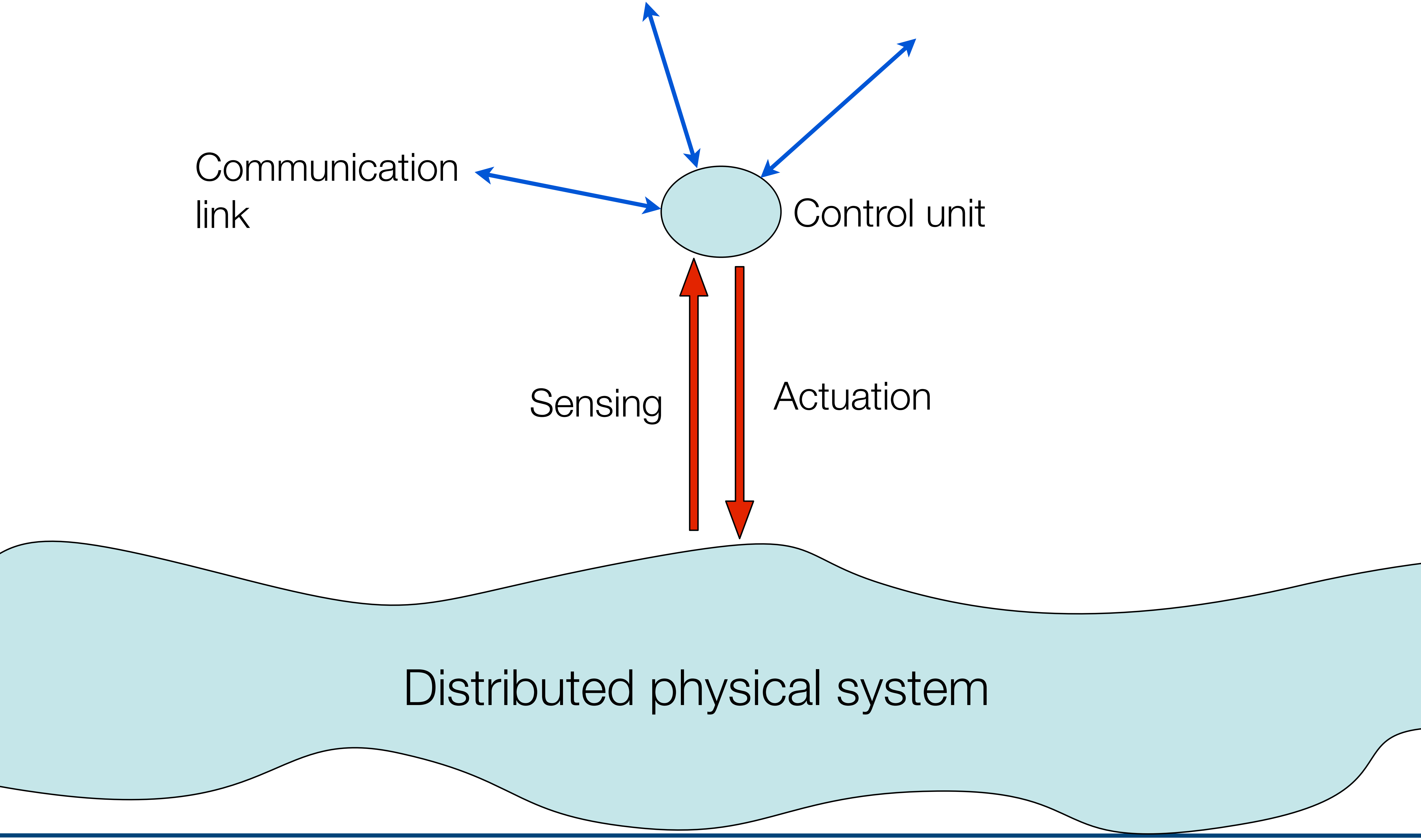
# Distributed decision models



Distributed control system

Distributed physical system

# Distributed decision models



# Example: robotic networks

Kiva systems

# Example: wireless sensor networks

# Distributed decision models

Water distribution

Traffic

# Leaderless distributed decision models

## Centralized vs. Leaderless

For large scale systems **centralized architectures** tends to be

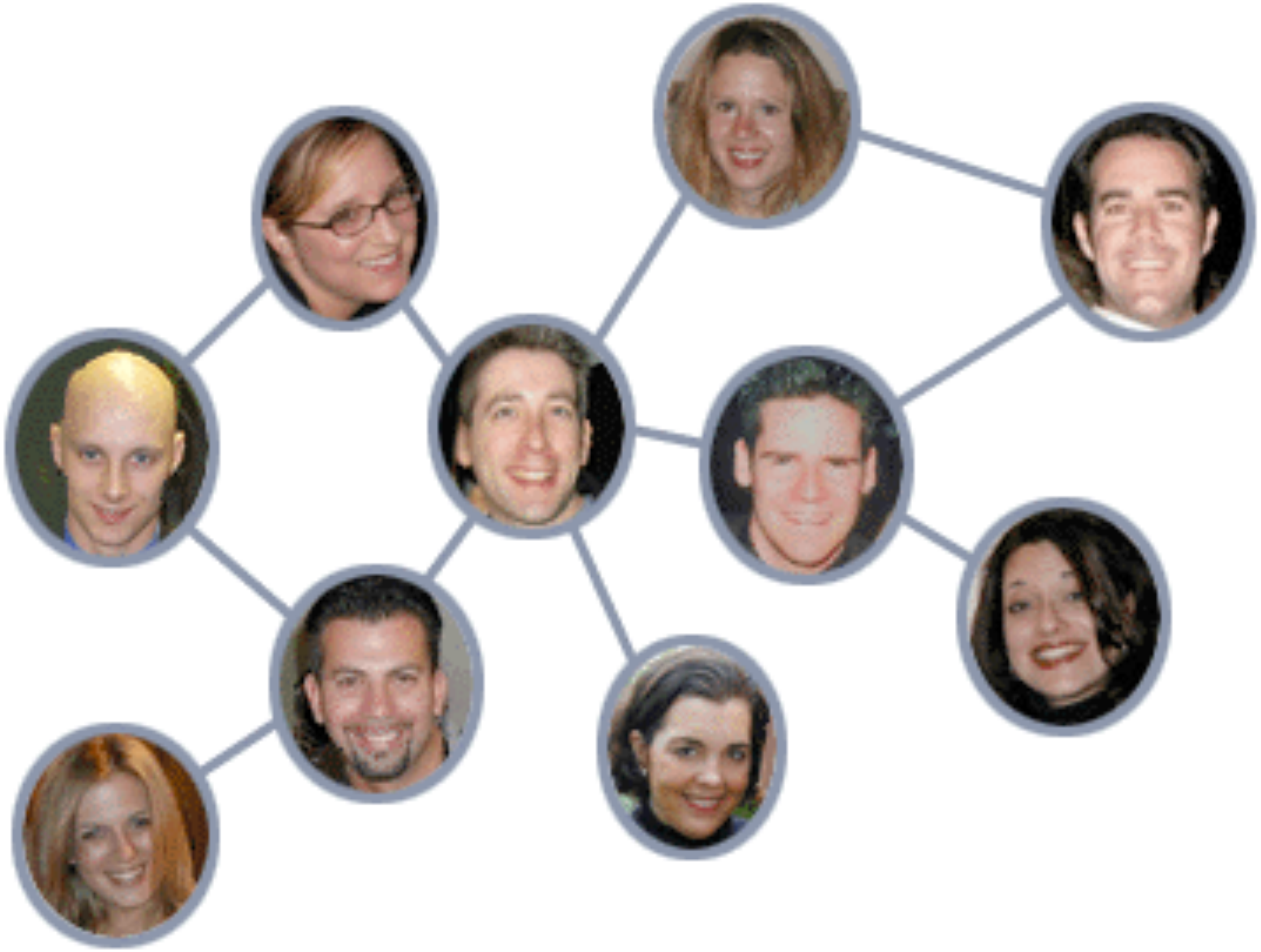
- **More efficient**
- **Fragile** to failures and to external changes
- **Expensive** in the configuration phase

For large scale systems **distributed architectures** tends to be

- **Less efficient**
- **Robust** to failures and to external changes (ex: market based economy)
- **Cheap** in the configuration phase (plug and play)

# Scientific context

Social and economic networks: individual social and economic interactions produce a global equilibrium **(market robustness)**



# Power distribution network

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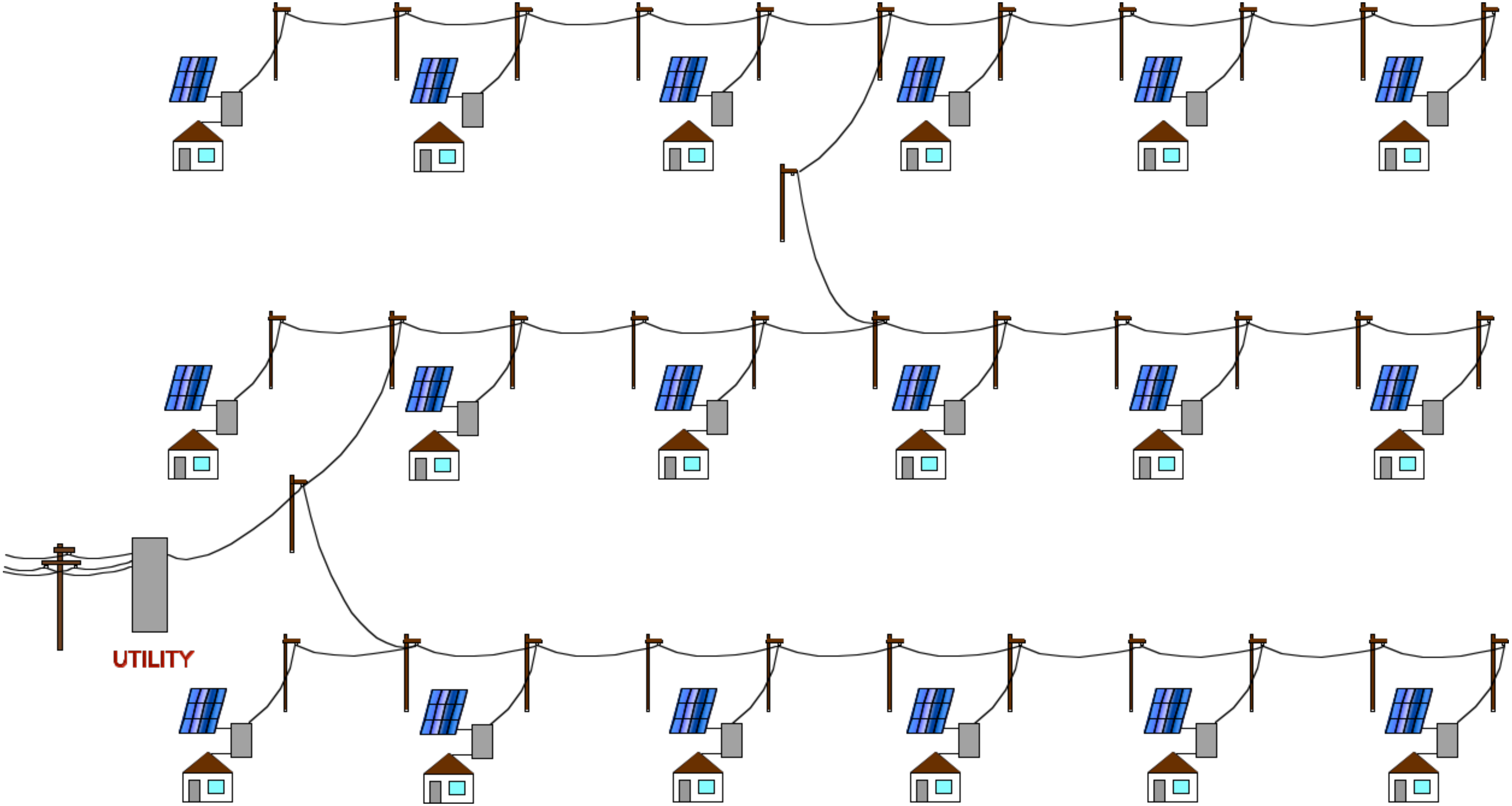
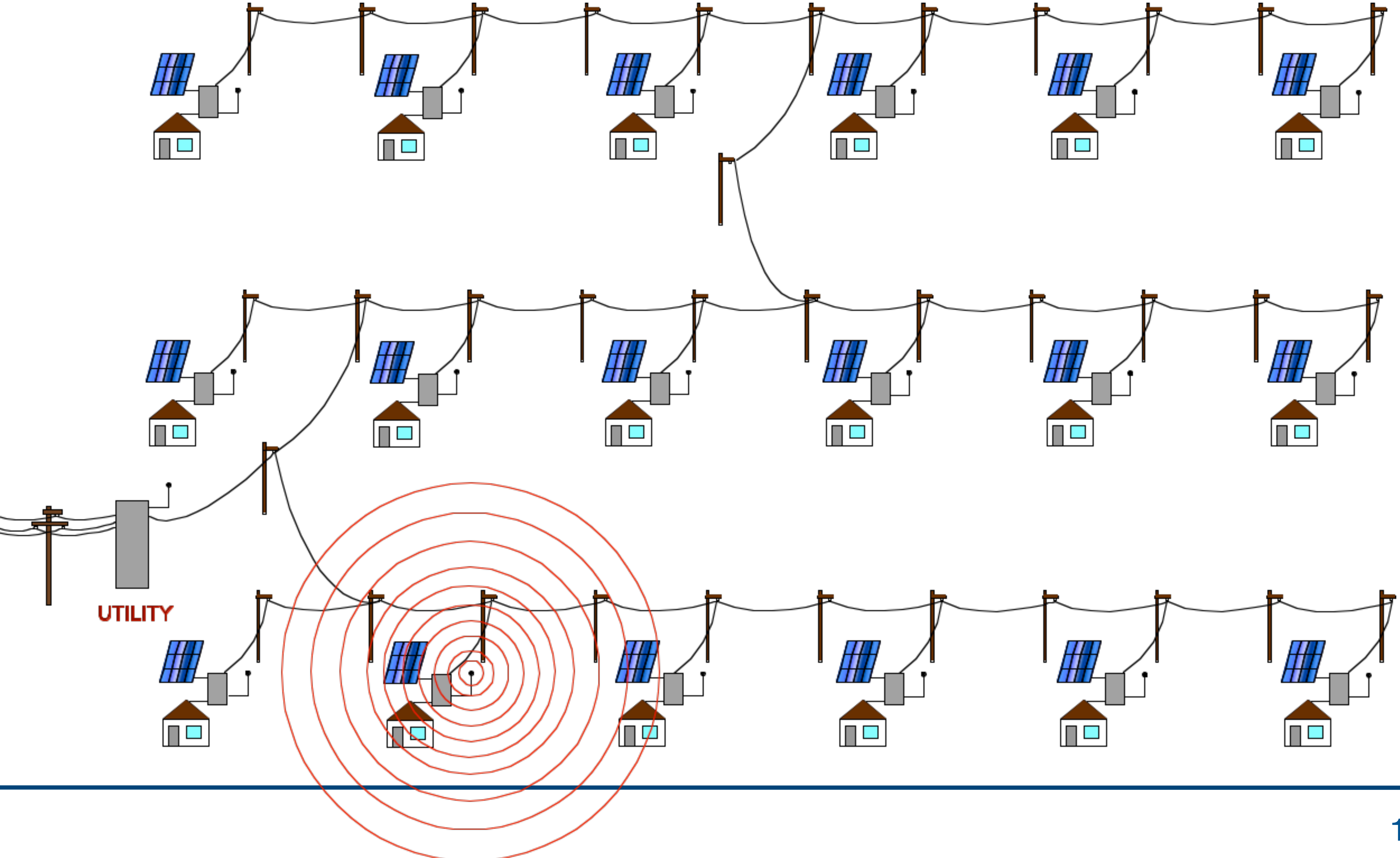
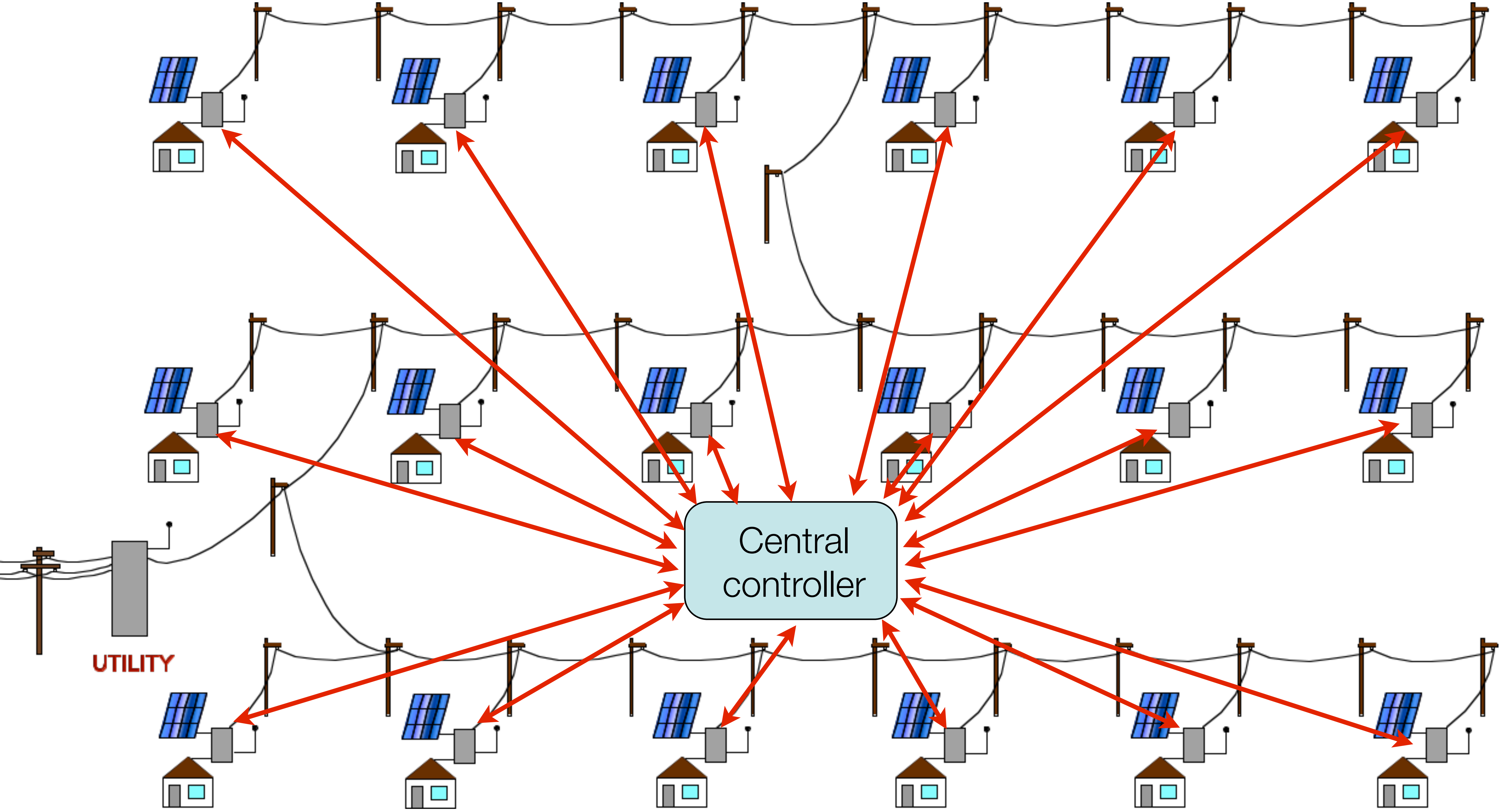


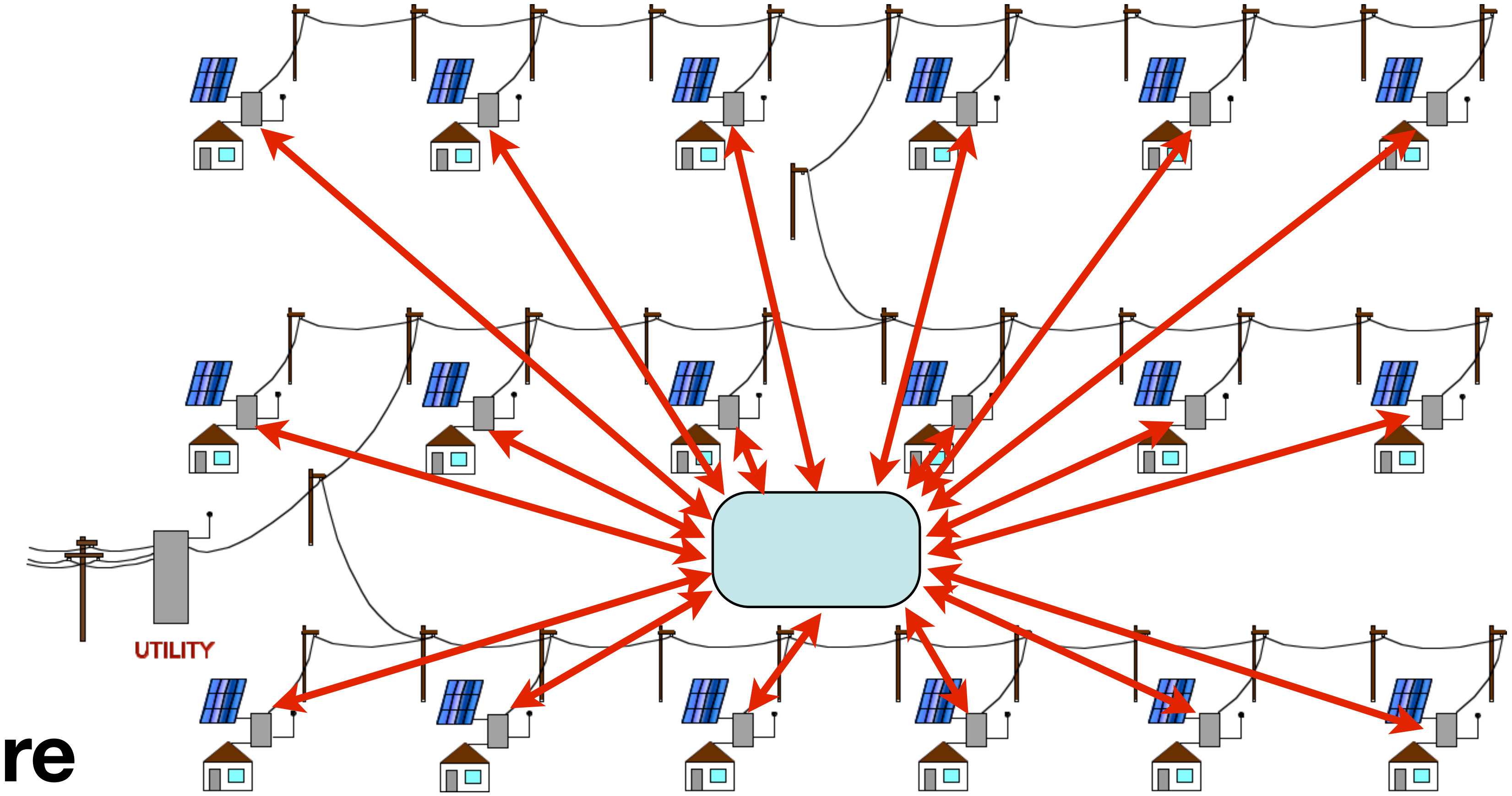
# Smart grid



# Centralized architecture

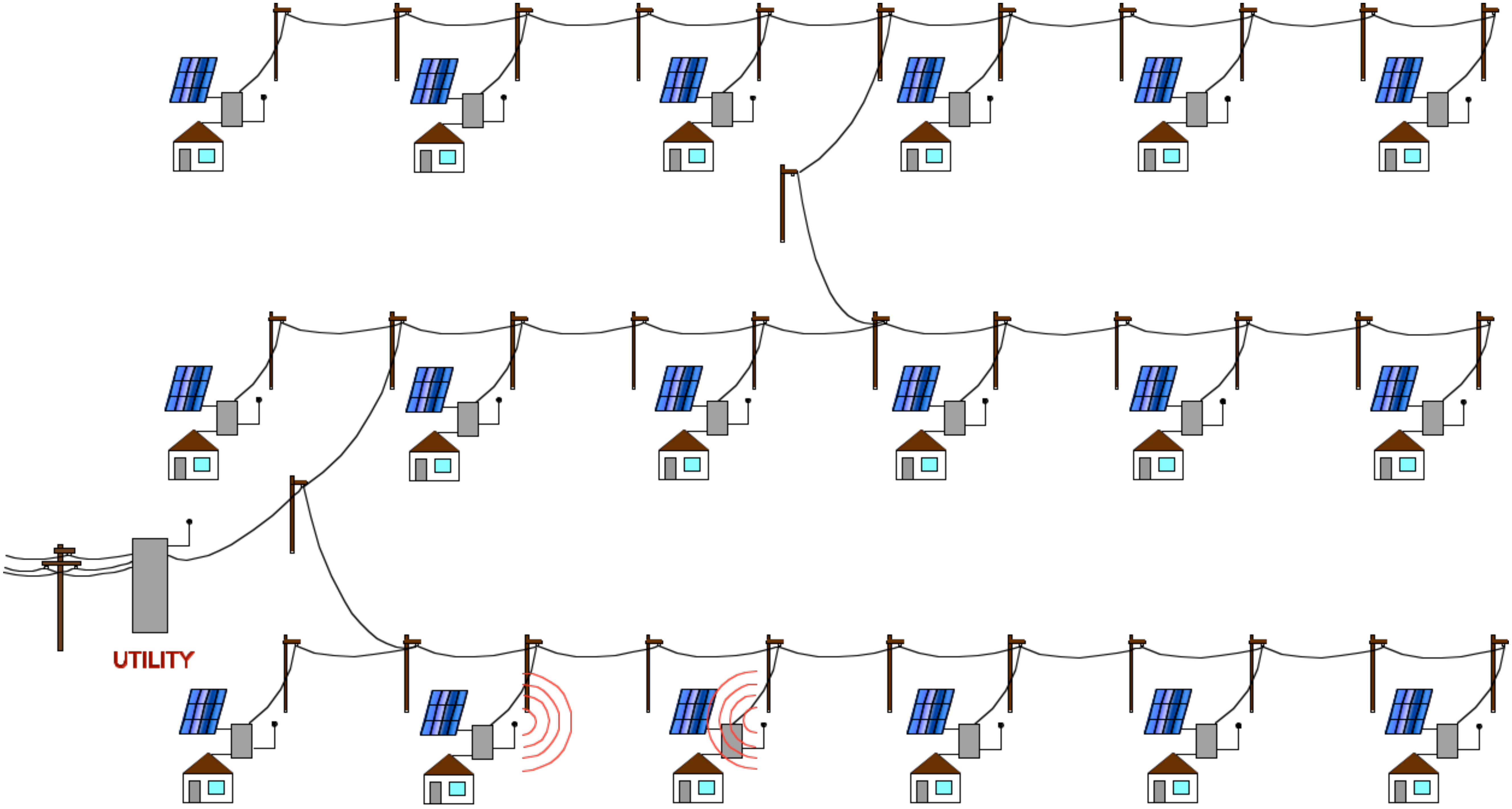


# Centralized architecture

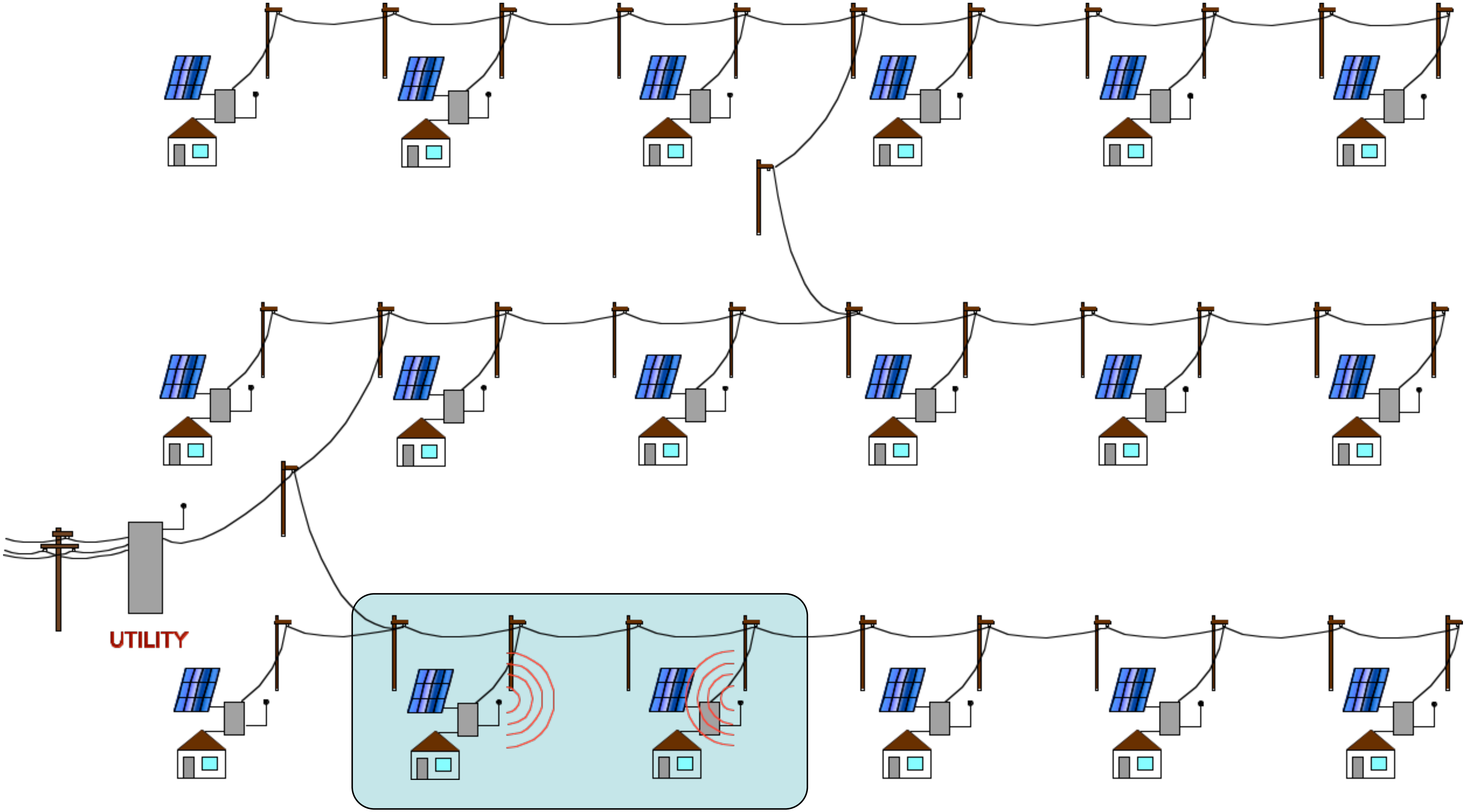


**Centralized architecture**
- **Efficient**
- **Local** sensing (voltage)
- **Local** control (injection of reactive power)
- **Global** and **synchronous** communication
- **Global** grid model
- **Expensive** configuration

# Distributed architecture



# Distributed architecture



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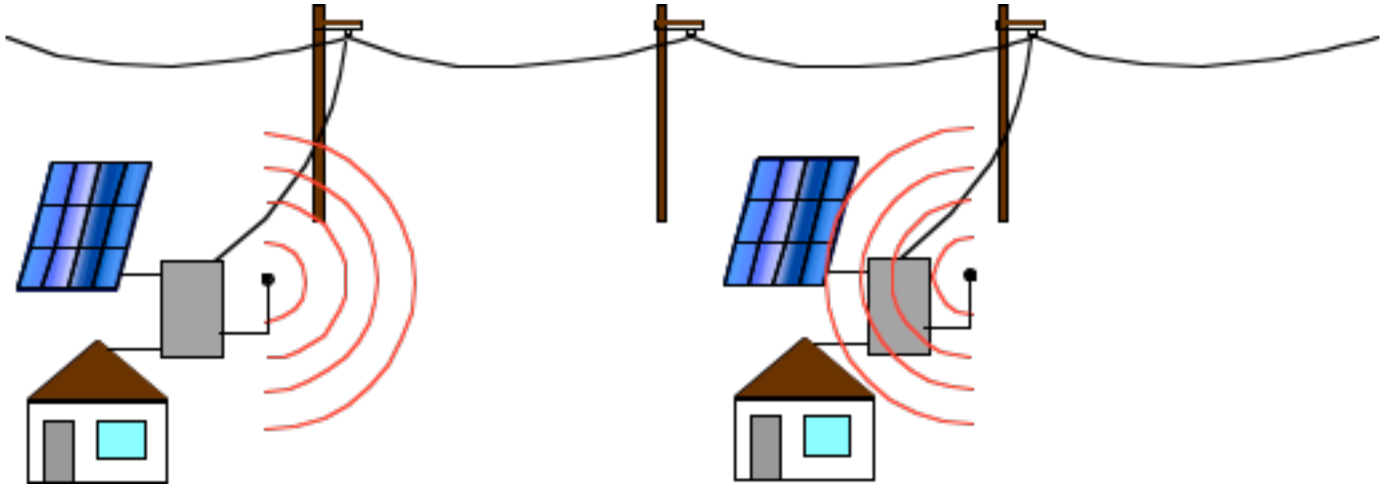

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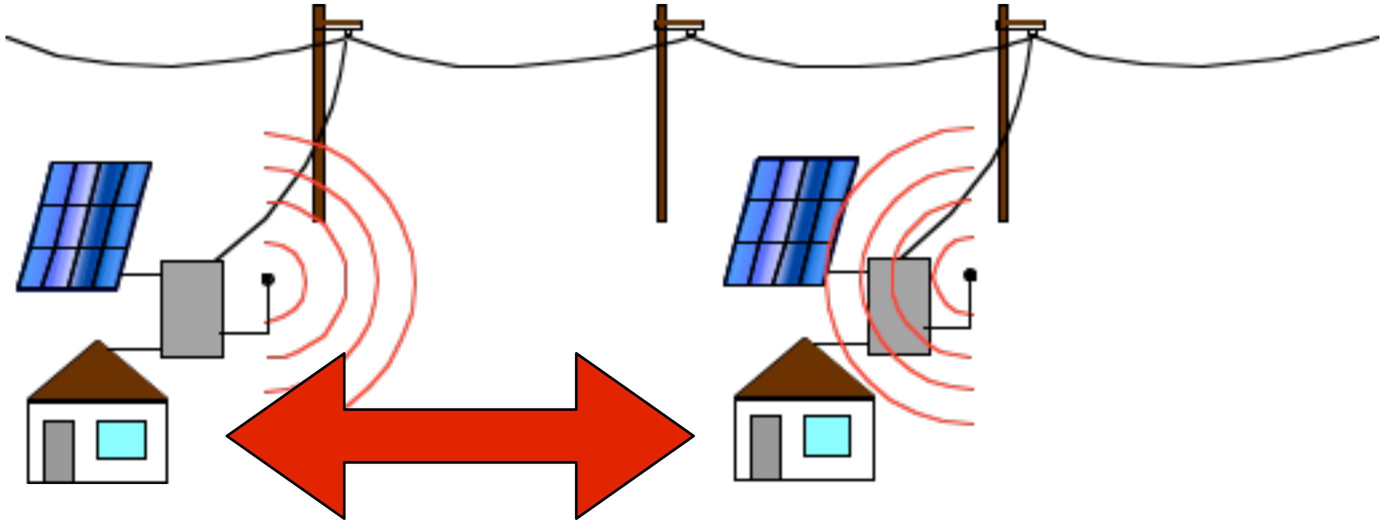

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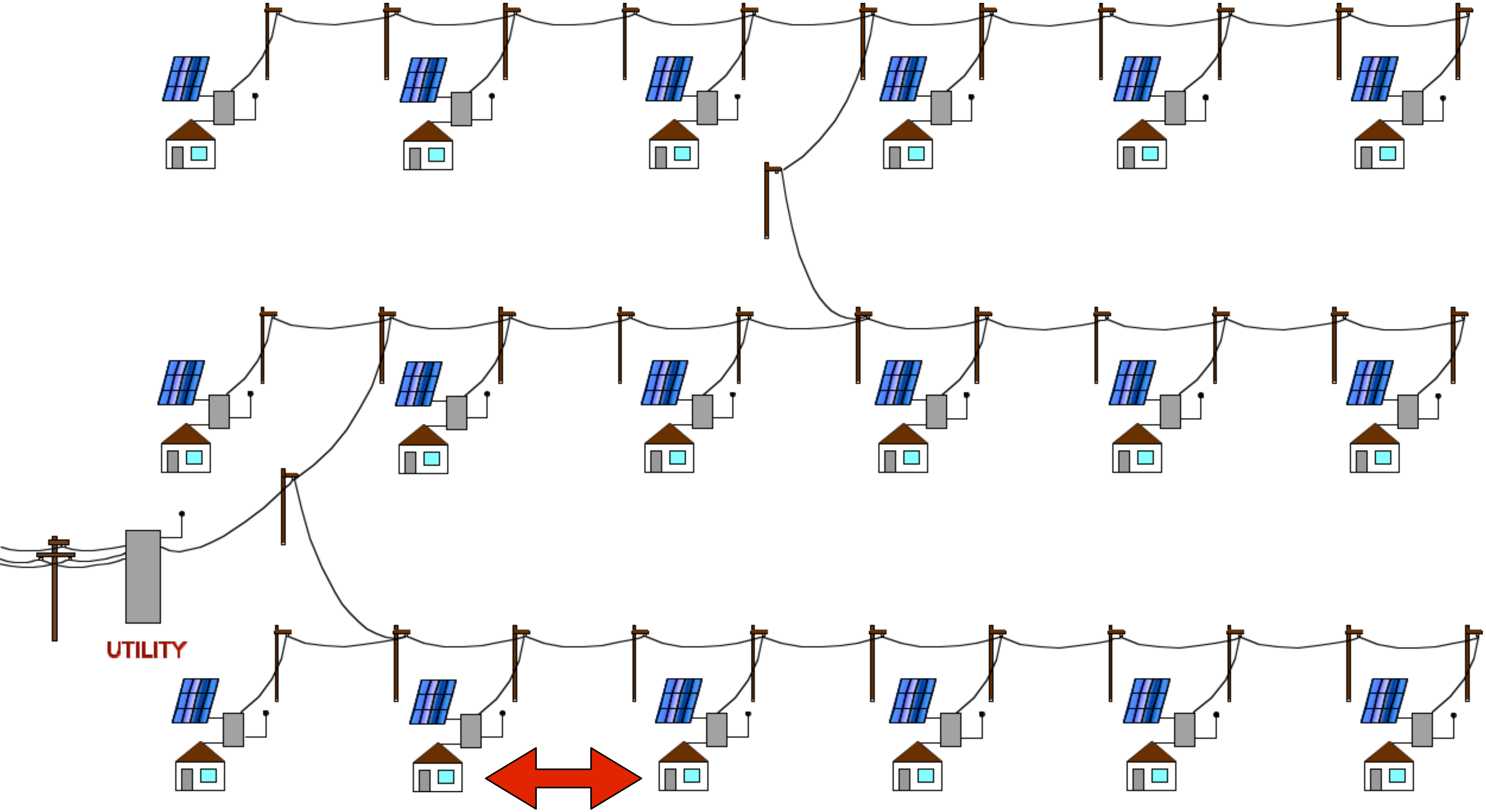

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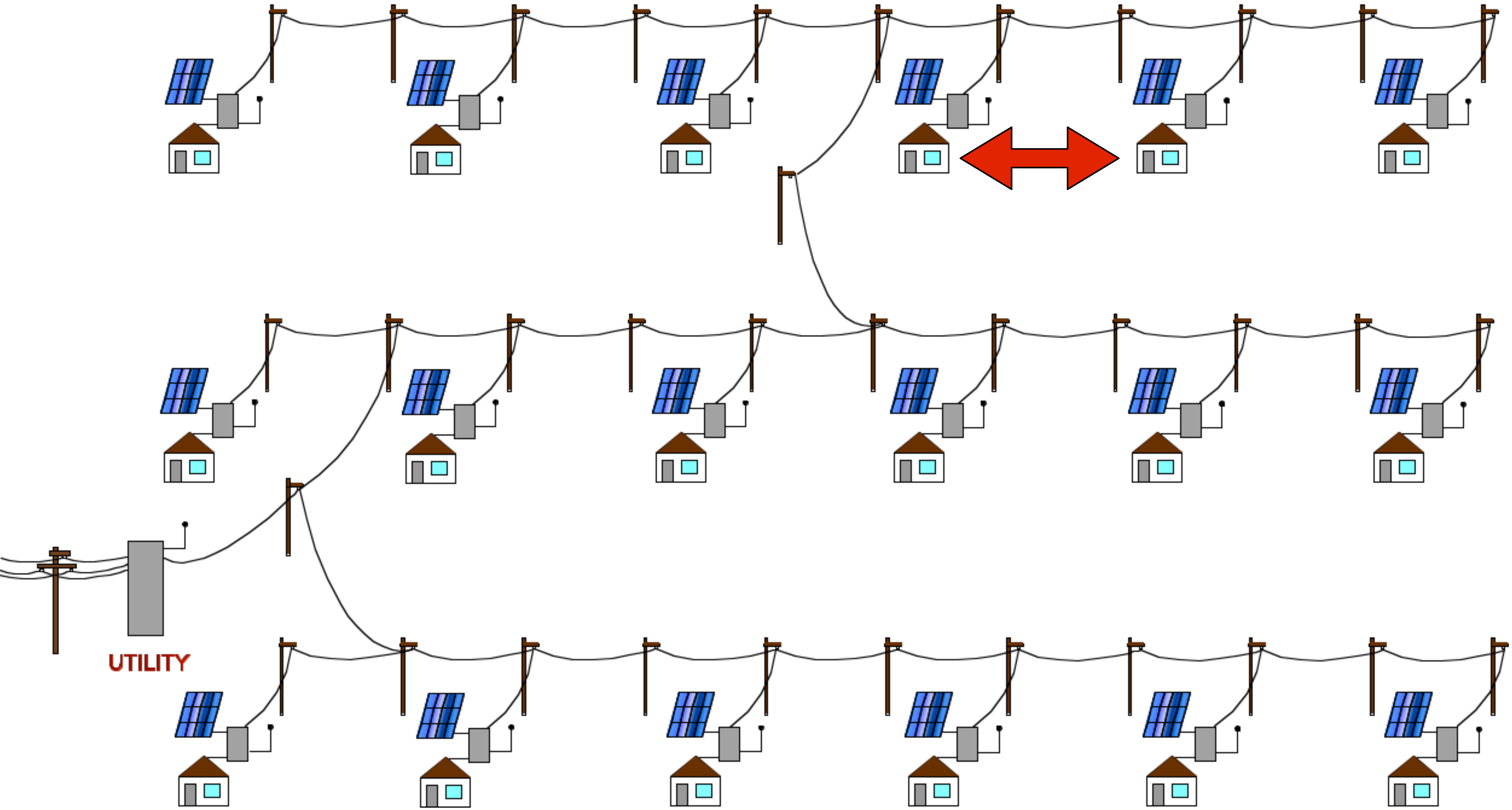

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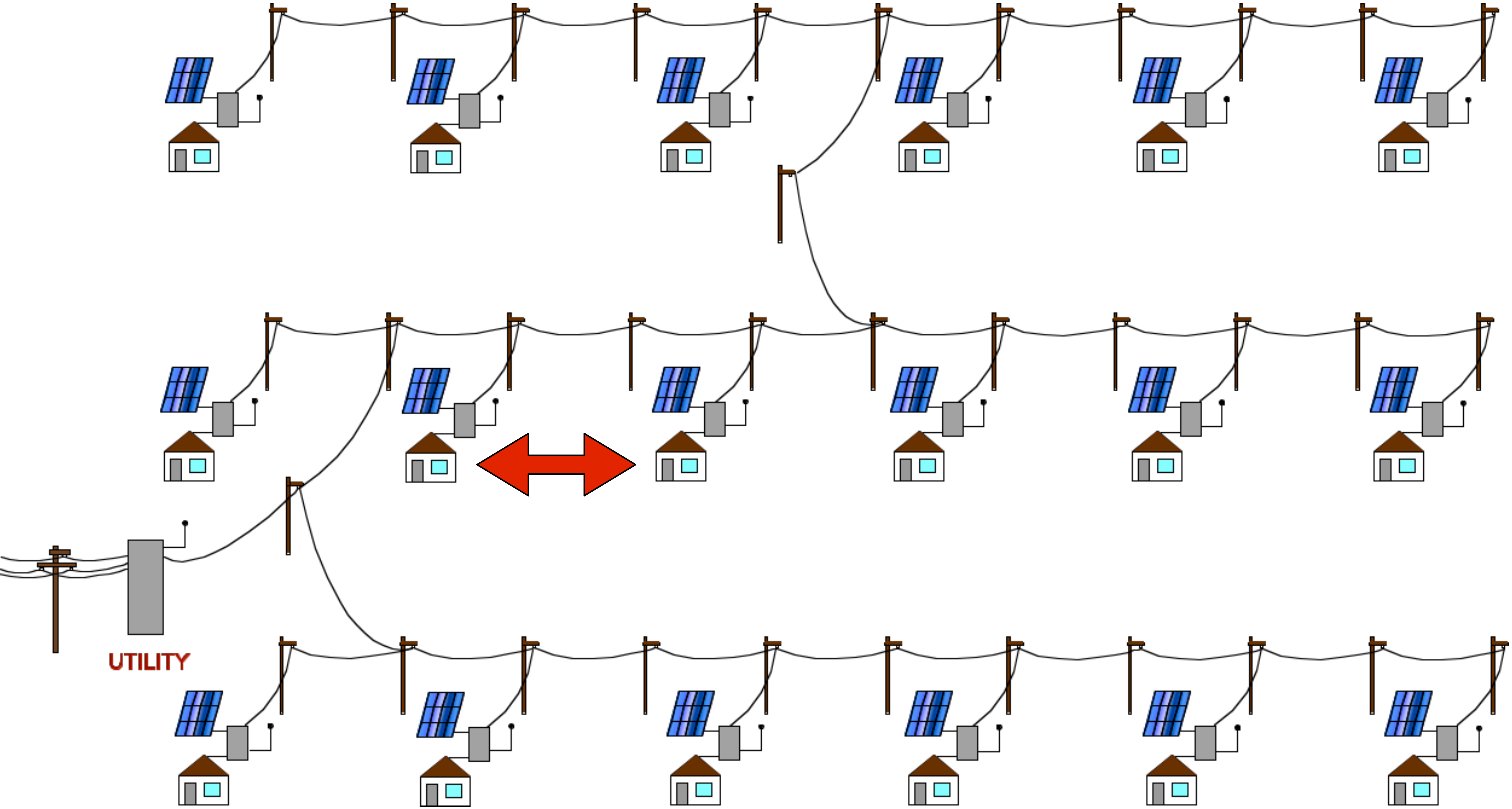

# Distributed architecture

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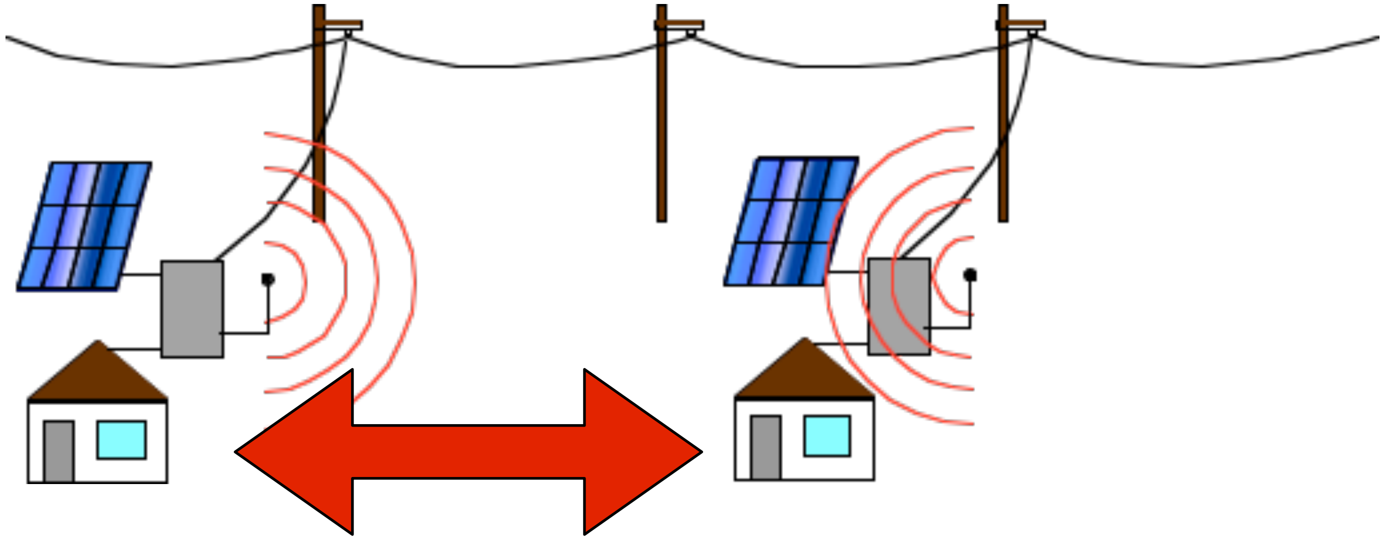


**Distributed leaderless architecture**

- **Less** efficient
- **Local** sensing (voltage)
- **Local** control (injection of reactive power)
- **Local** and **asynchronous** communication
- **Local** grid model
- **Cheap** configuration

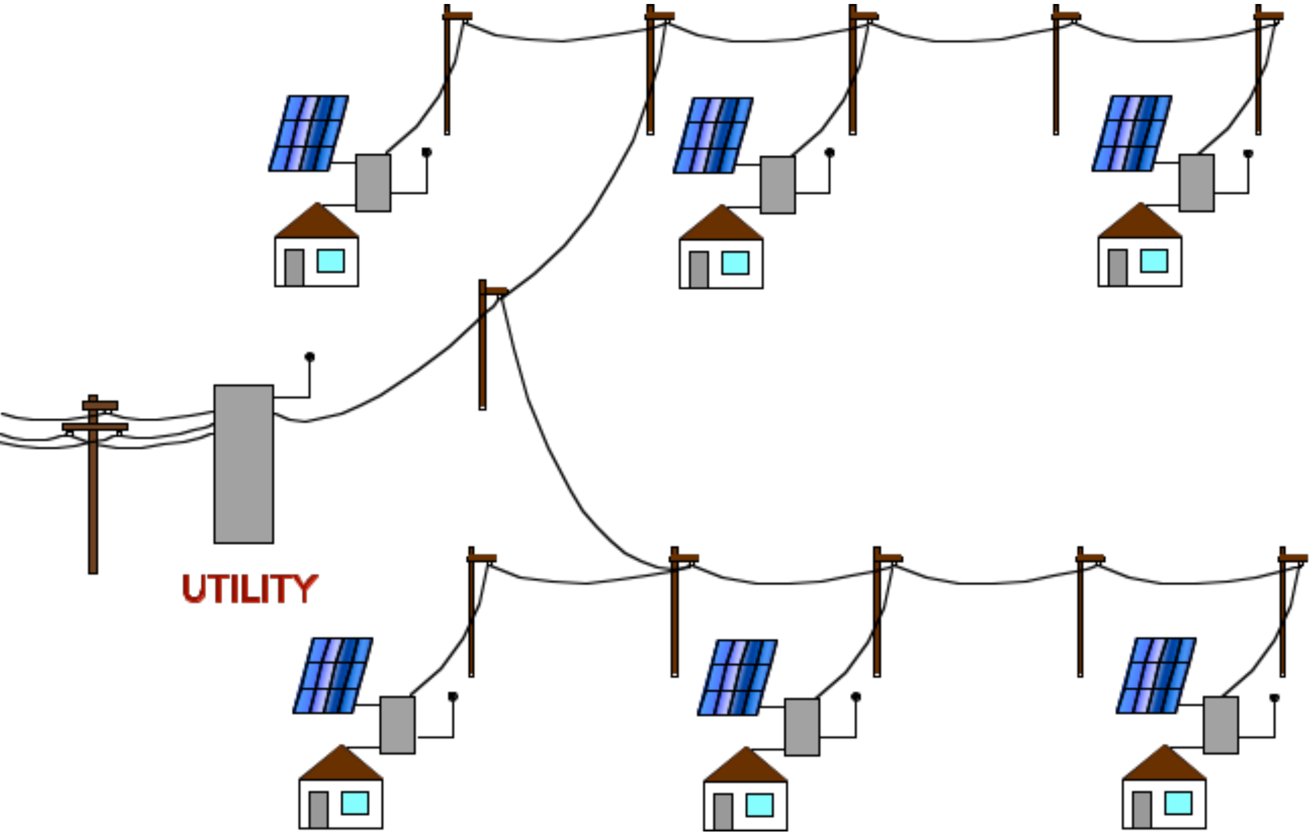
# The reactive power

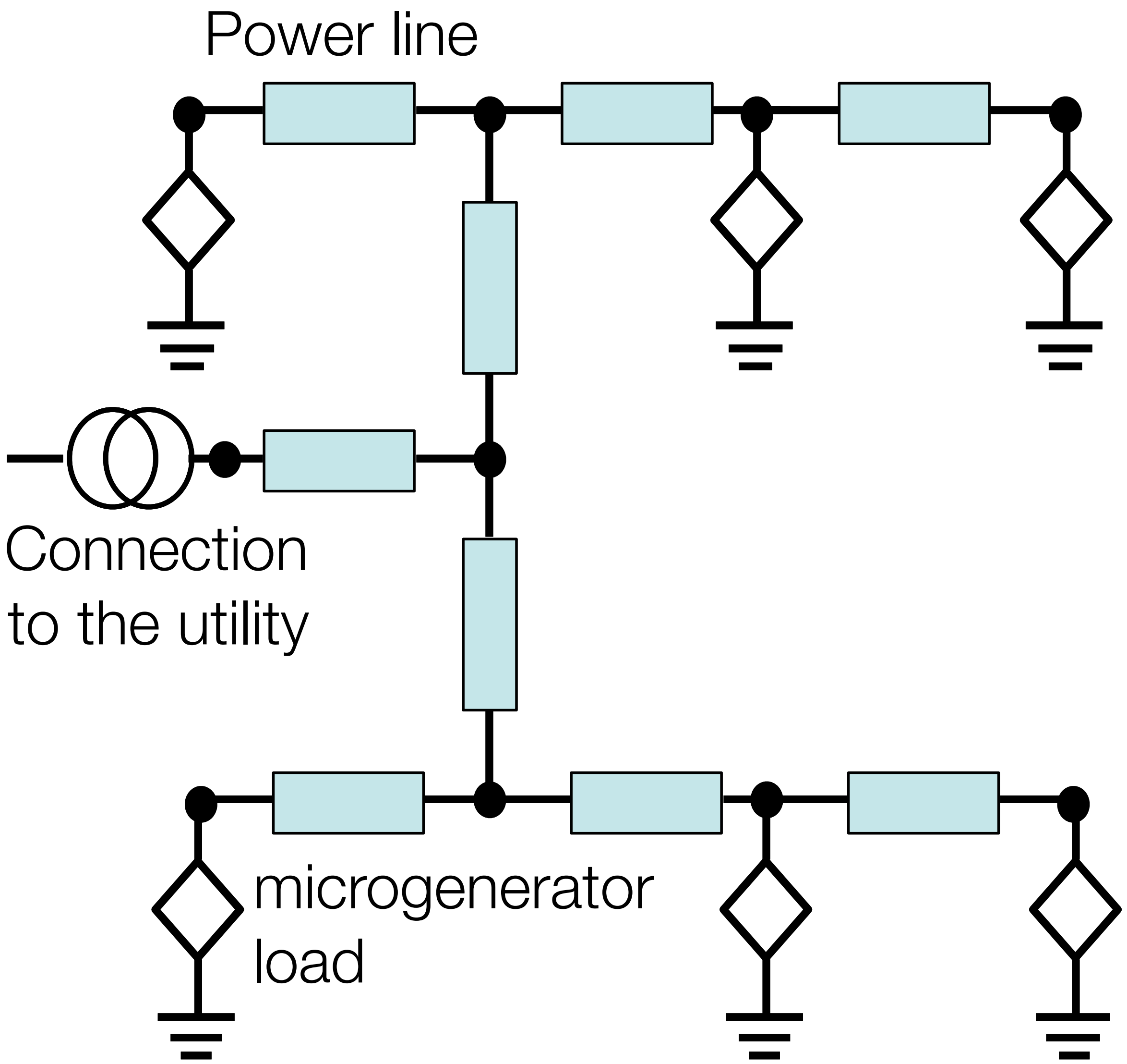
We have reactive power whenever voltage and current are out of phase, i.e. phase angle is not zero.

- Users in the microgrid may require reactive power
- It can be obtained from the utility which in this case charges the microgrid
- It can produced by the electronic interfaces of microgenerators in the microgrid with (essentially) no cost
- Transporting reactive power costs since it yields losses on the cables

Consequently it is convenient to generate reactive power close where it is needed

# A model of a microgrid

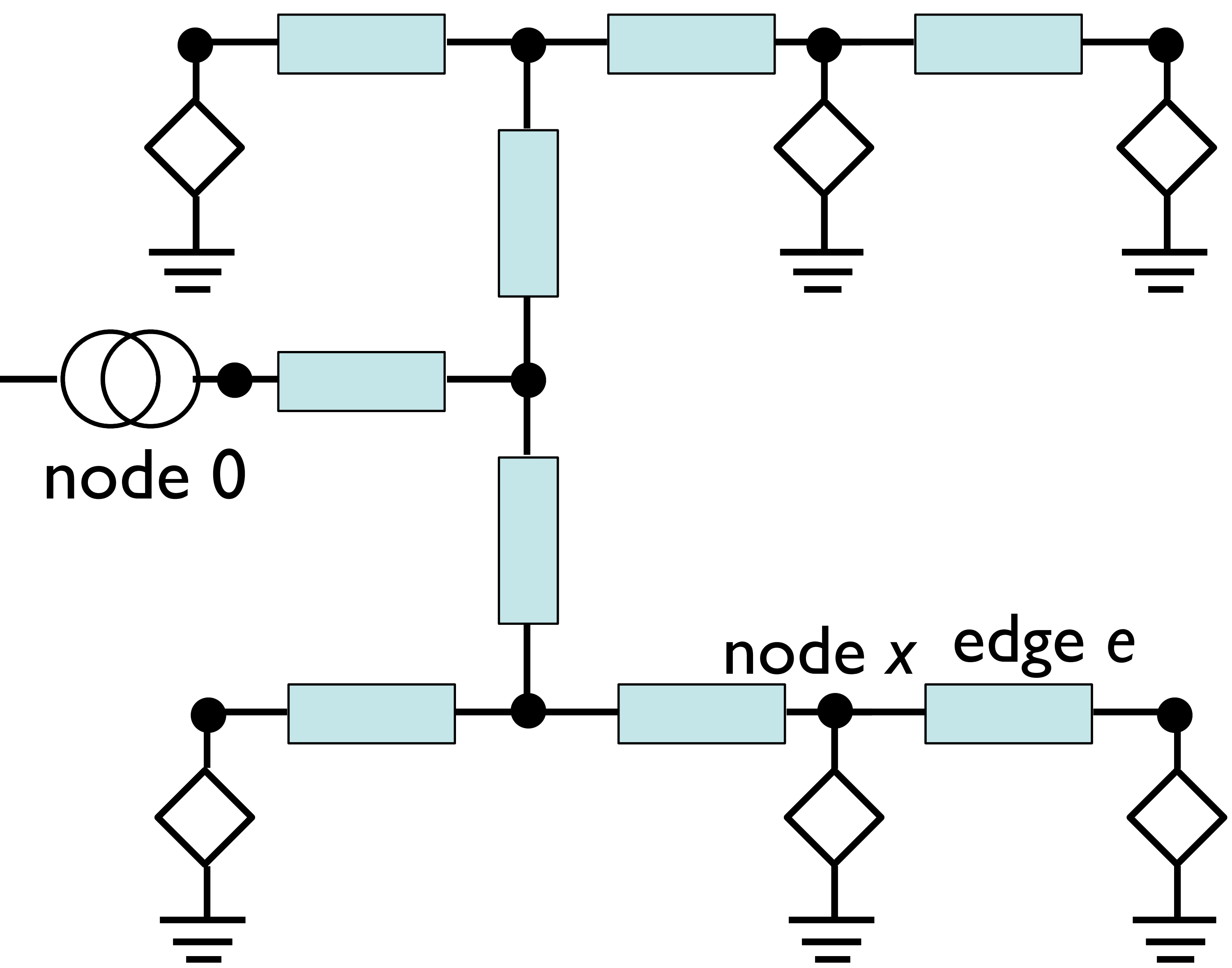




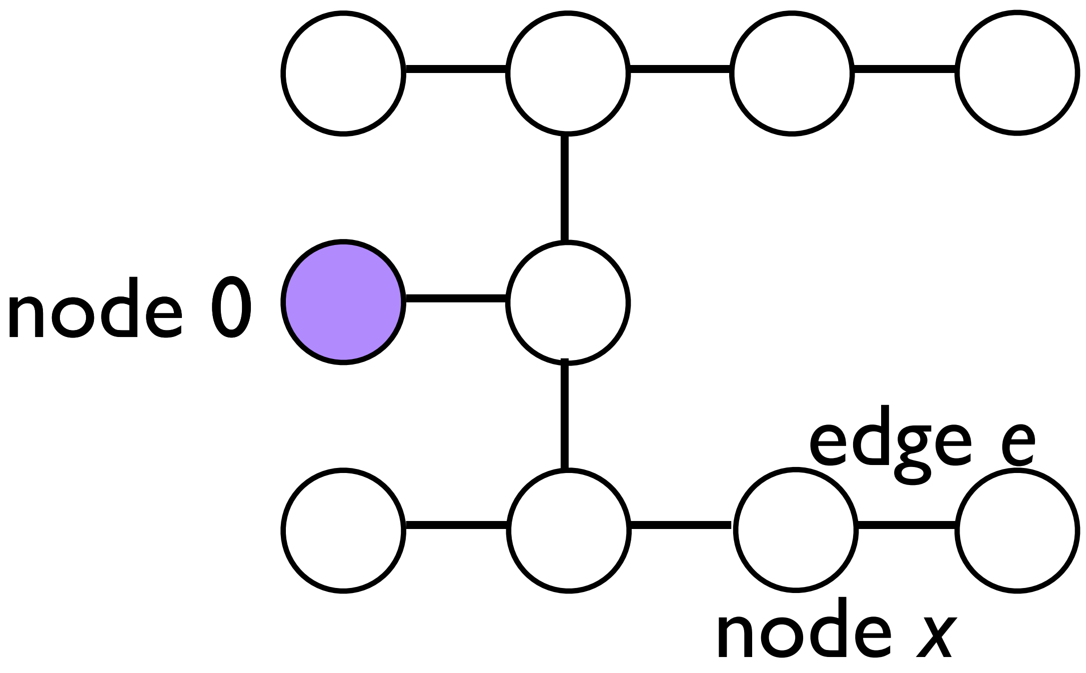
Power lines: impedences i.e. linear constraints on currents and voltages

Microgenerators/loads: linear constraints in the (active and reactive) powers

# A model of a microgrid



Graph model



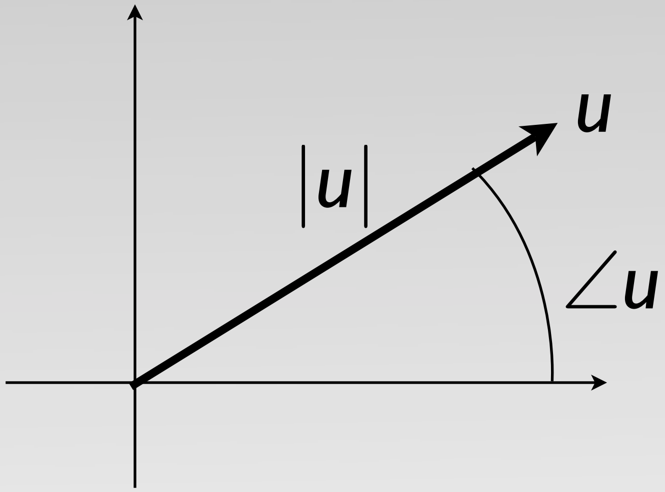
# A model of a microgrid

## Sinusoidal regime

We assume that the circuit is at the sinusoidal regime at a certain fixed frequency. In this way every signal $u(t)$ are described by a complex number $u \in \mathbb{C}$ describing amplitudes and phases

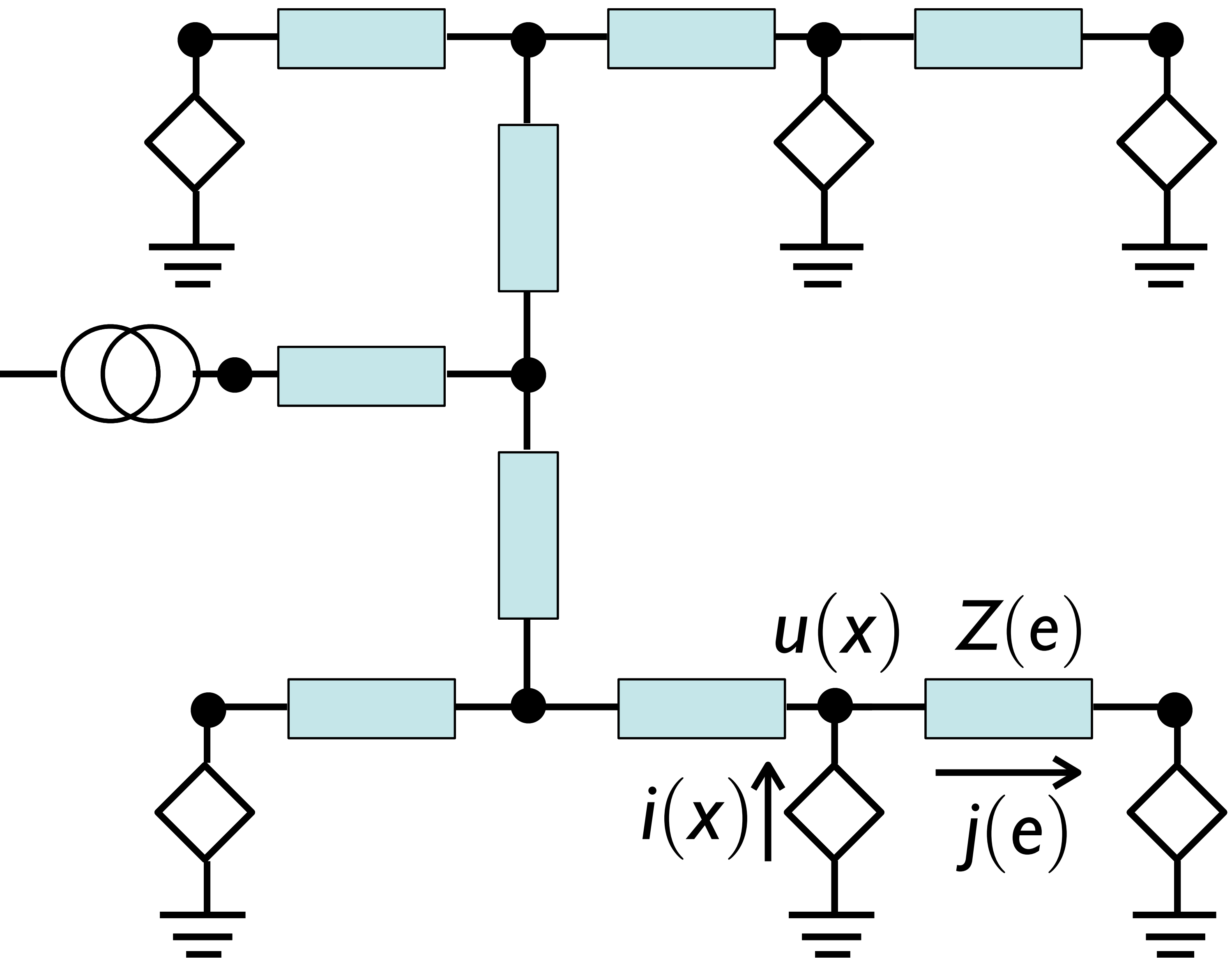
$$u(t) = |u| \cos(\omega t + \angle u)$$

where $|u|$ is the absolute value of $u$ and $\angle u$ is the phase of $u$.



$$u = |u| e^{j \angle u}$$

# A model of a microgrid



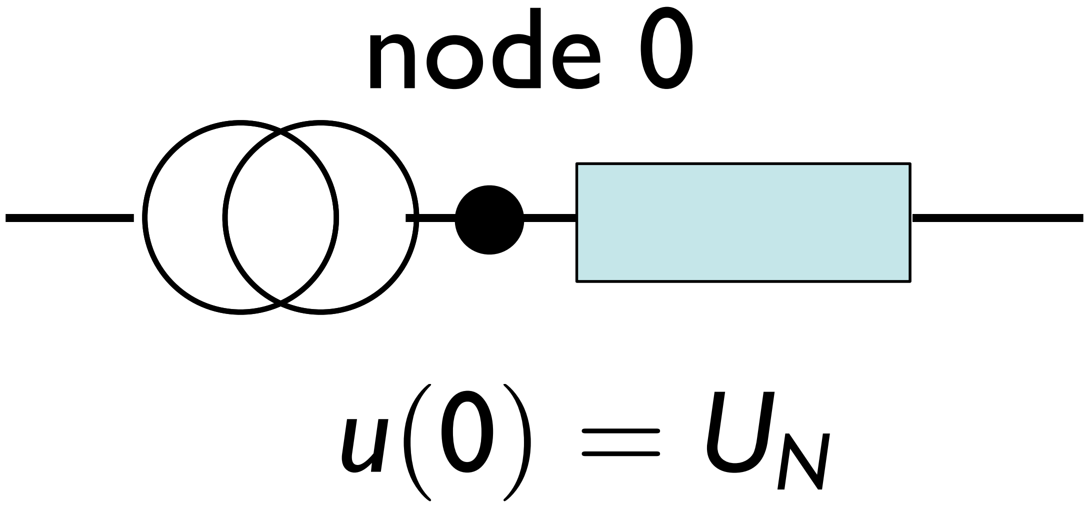
## Variables of the model

$u(x)$ potential at node $x$
$i(x)$ current at node $y$
$j(e)$ current at edge e
$Z(e)$ impendence at edge e

complex numbers

# A model of a microgrid

The utility ensures that the voltage at the node 0 is equal to the nominal voltage $U_N$



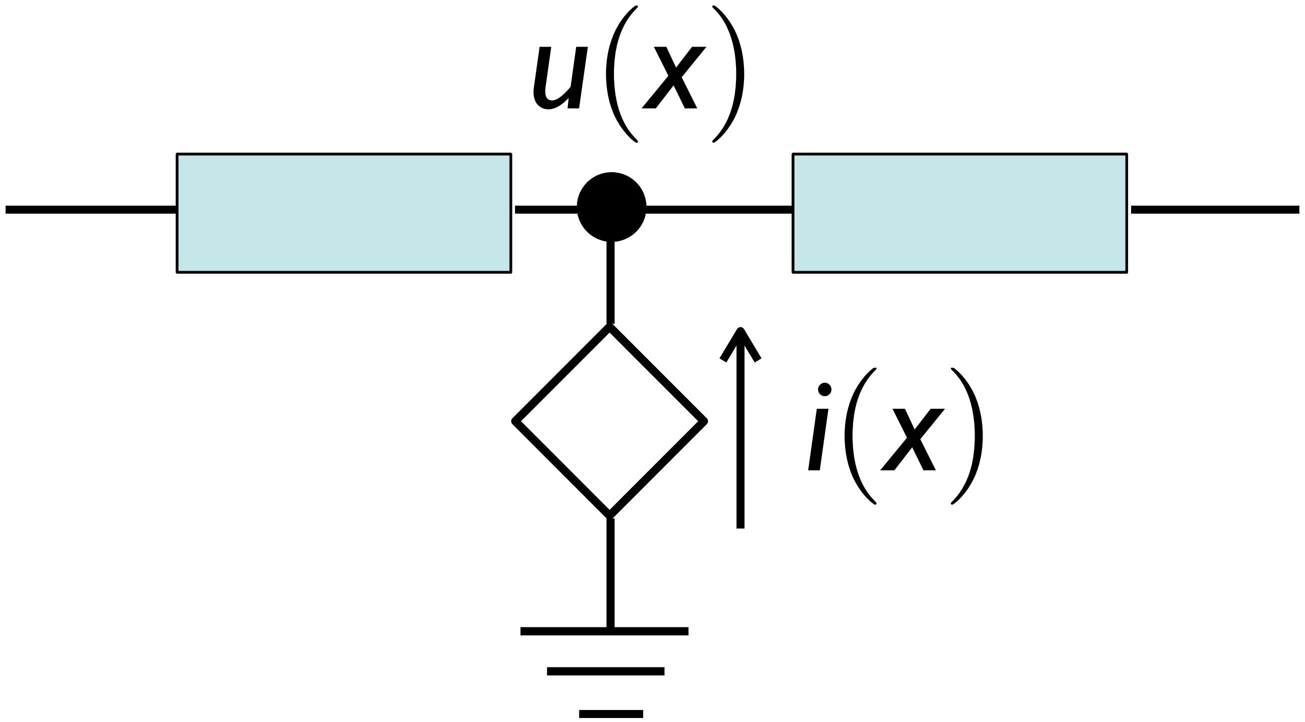
$$u(0) = U_N$$

# A model of a microgrid

The node $x$ inject in the grid the complex power $s(x) = u(x)i(x)^*$

$p(x) = \mathrm{Re}[s(x)]$ active power

$q(x) = \mathrm{Re}[s(x)]$ reactive power



# A model of a microgrid

The cost to be minimized is the Power Loss $(\mathrm{PL})$

$$\mathrm{PL} = \sum_{e} \mathrm{Re}[Z(e)]|j(e)|^2$$

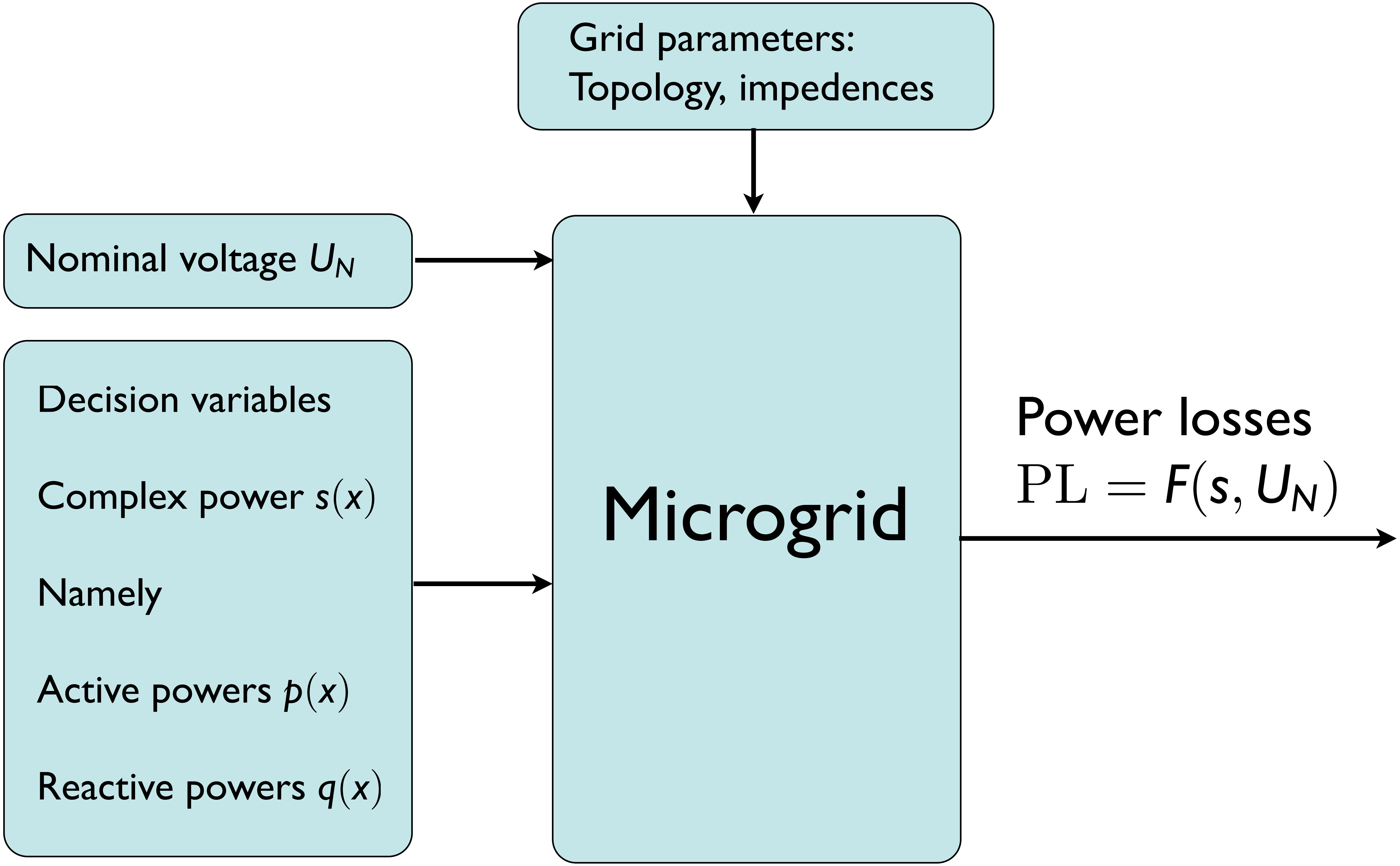
The inputs (control/disturbances)

- $U_N$ (nominal voltage)
- $s$ (the vector having as entries the injected complex powers $s(x)$)

The cost $\mathrm{PL}$ is a nonlinear function of the imputs

$$\mathrm{PL} = F(s, U_N)$$

# A model of a microgrid



Grid parameters:
Topology, impedences

Nominal voltage $U_N$

Decision variables

Complex power $s(x)$

Namely

Active powers $p(x)$

Reactive powers $q(x)$

Microgrid

Power losses
$\mathrm{PL} = F(s, U_N)$

# Minimization of the power losses

$$\min_{\text{over } s} \quad F(U_N, s)$$

subject to constraints

**NON-CONVEX OPTIMIZATION PROBLEM: difficult to solve in a distributed way**

# Approximation of the cost

**HYPOTHESIS:** $Z(e) = e^{j\theta} R(e)$ where $R(e)$ is a real number

**Taylor Expansion**

$$F(s, U_N) \simeq s^* M s \, \frac{1}{U_N^2} + o\left(\frac{1}{U_N^2}\right)$$

For big $U_N$, minimizing $F(s, U_N)$ is equivalent to minimizing the quadratic function

$$\boxed{s^* M s}$$

# Approximation of the cost

HYPOTHESIS: $Z(e) = e^{j\theta} R(e)$ where $R(e)$ is a real number

**Approximation of the gradient of the cost function**

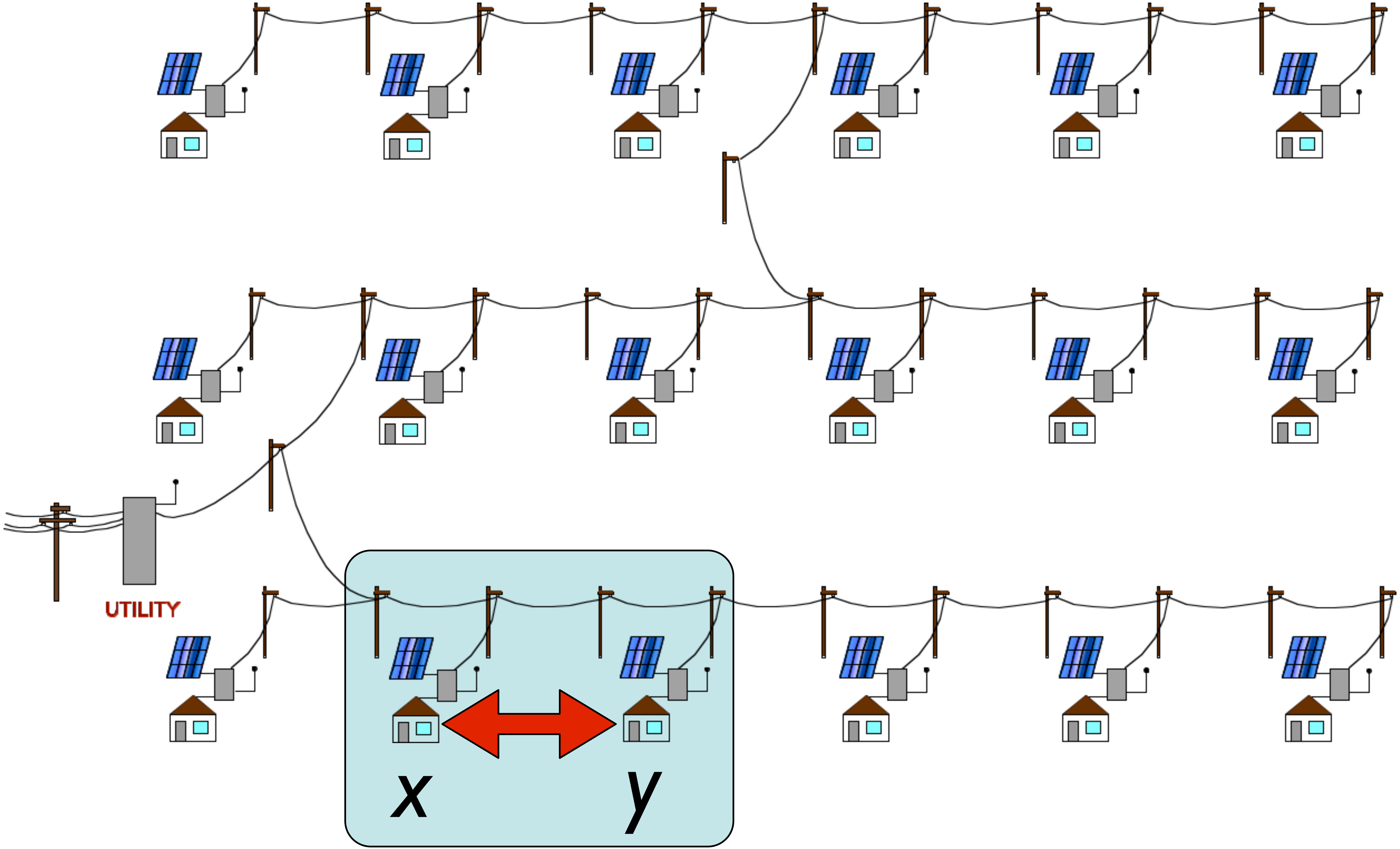
If $U_N$ is big, then

$$\text{grad}\, F(s, U_N) \simeq 2 \frac{\cos\theta \; e^{j\theta}}{U_N} (u - U_N \mathbf{1})^*$$

The gradient of $F(s, U_N)$ can be obtained by measuring the voltages

# Distributed Iterative Algorithm

Iterative algorithm: at each step *t* a random pair of nodes *x*, *y* are activated



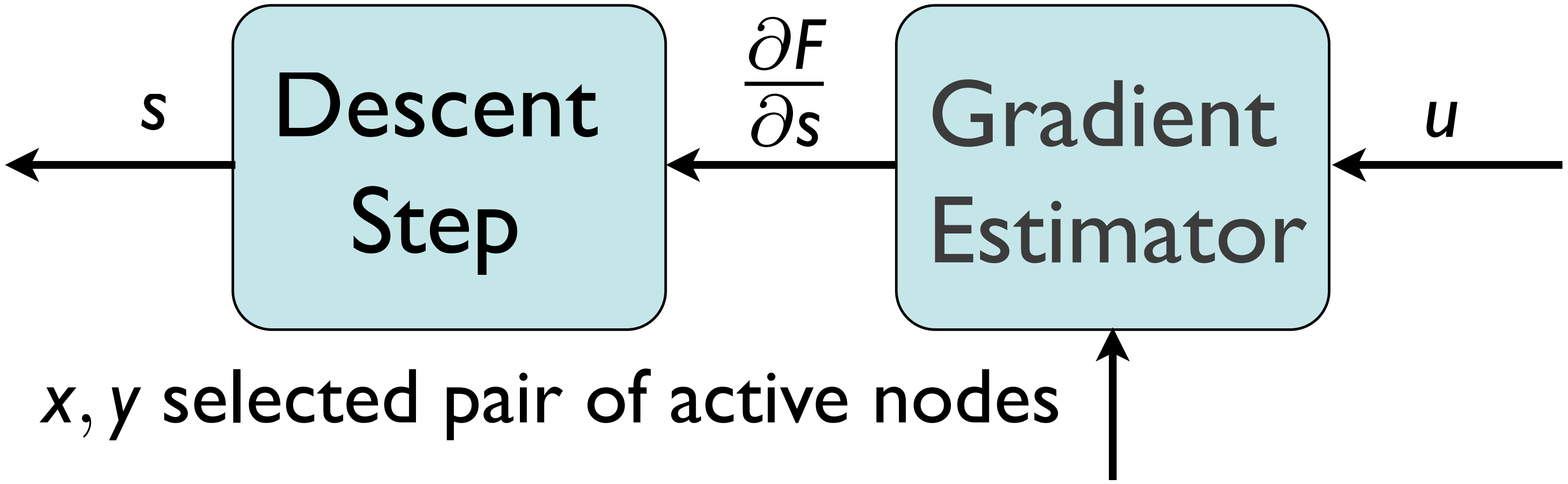
# Distributed Iterative Algorithm

## Estimation step

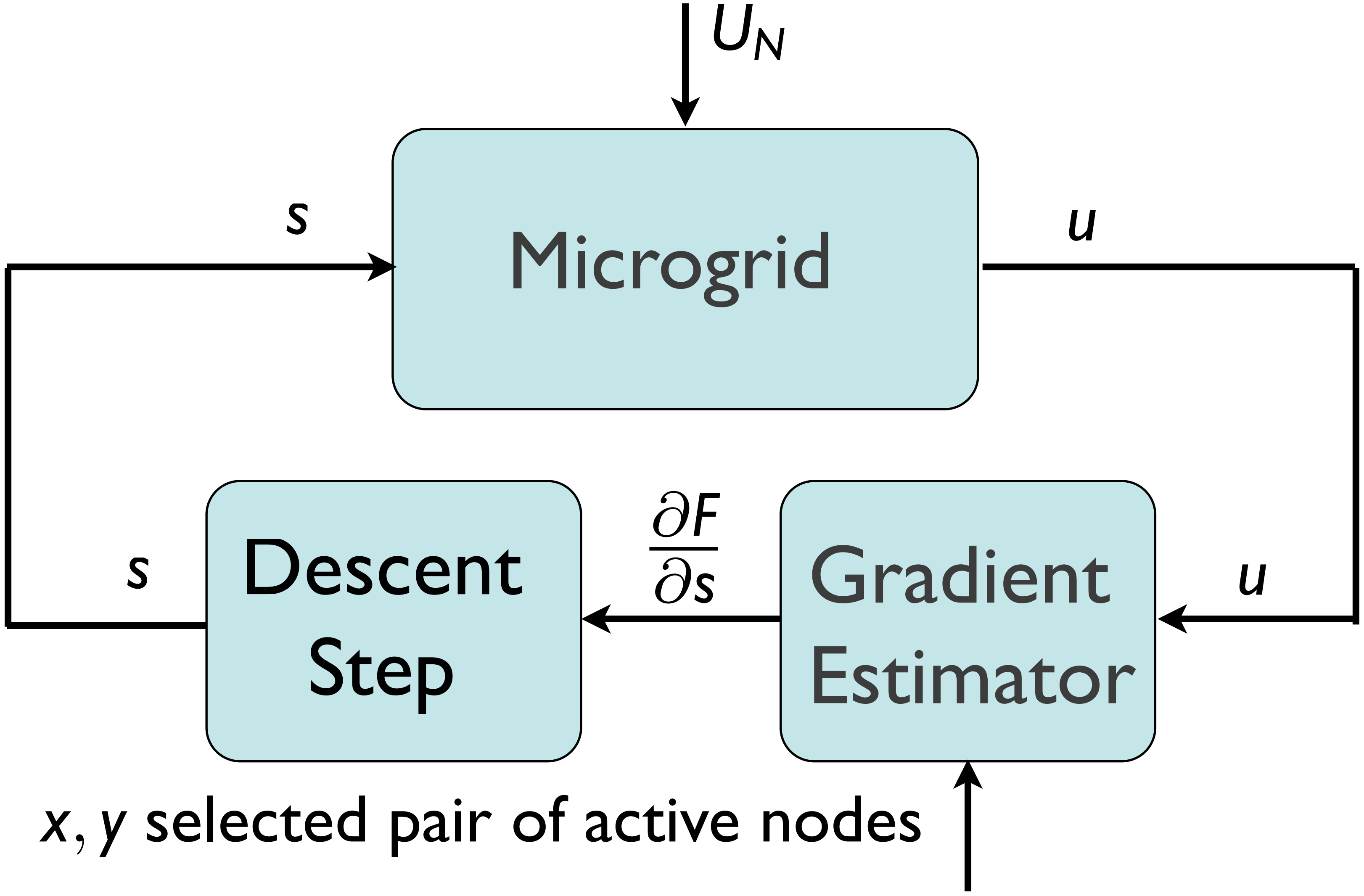
The nodes $x, y$ estimate the gradient at $x, y$ from the voltages $u(x), u(y)$ and exchange these estimates

## Descent step

The nodes $x, y$ inject the new powers $s(x), s(y)$ so that the cost decreases

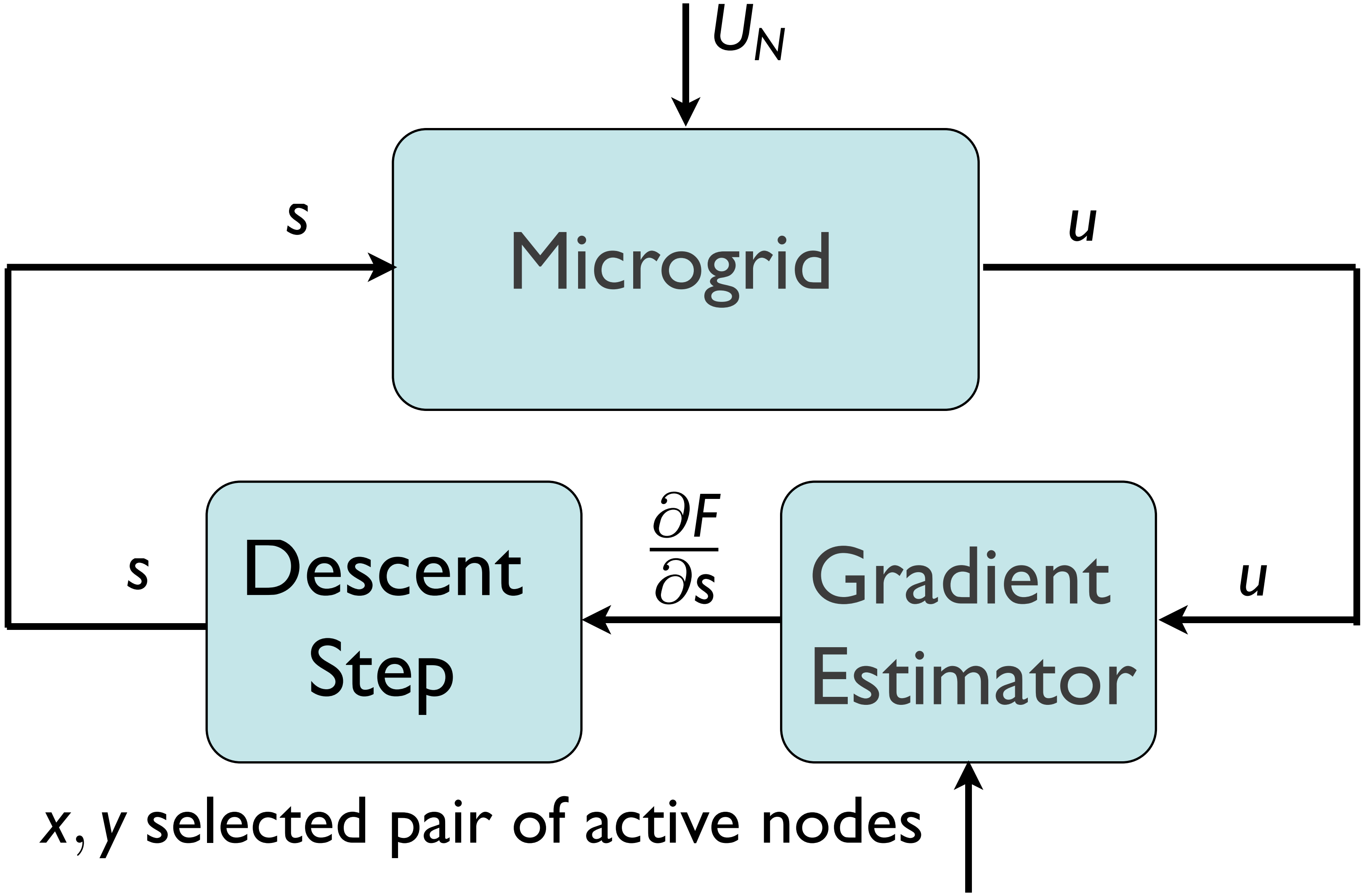


# Distributed Iterative Algorithm



$x, y$ selected pair of active nodes

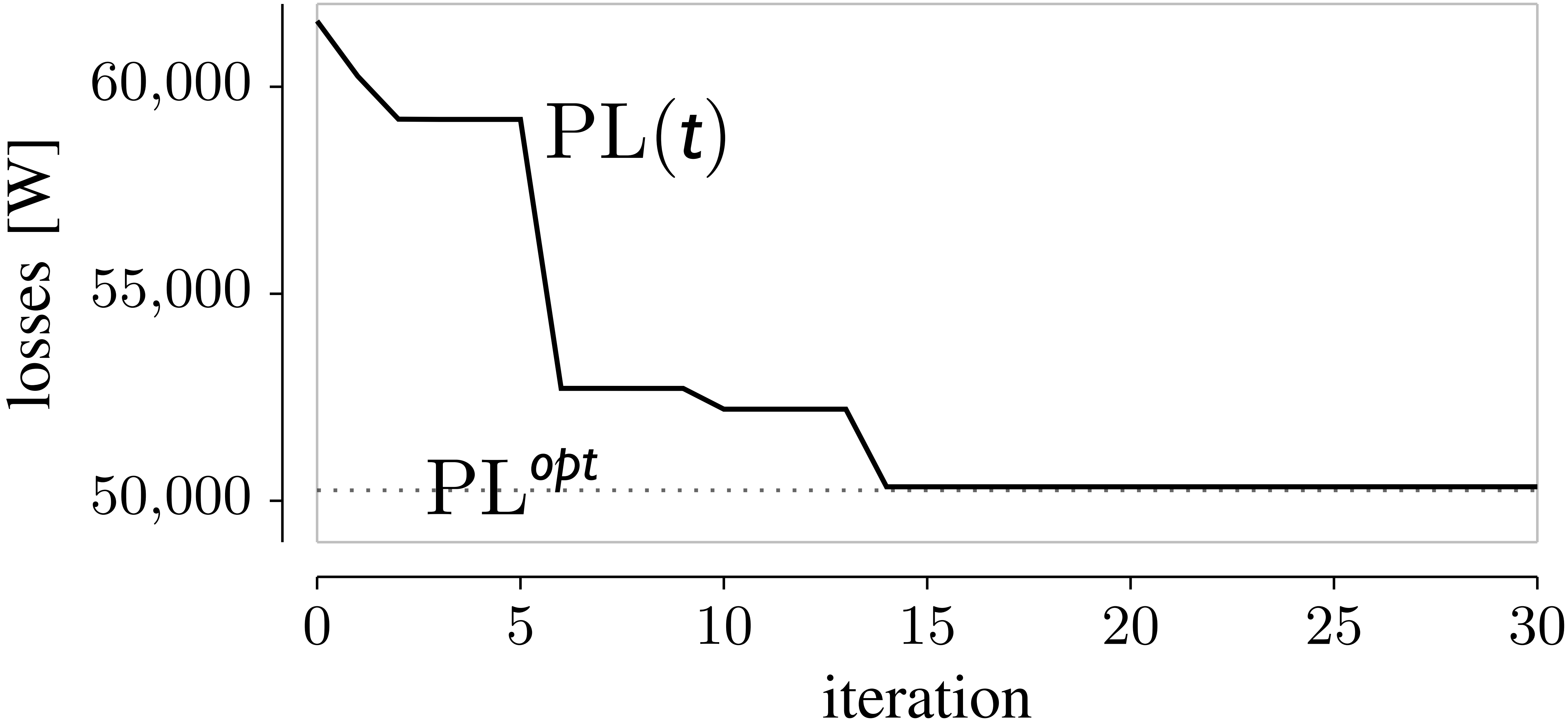
# Distributed Iterative Algorithm



**Result:** the power loss converges to an approximation of the optimal power loss

$$\mathrm{PL}(t) \longrightarrow \mathrm{PL}^{opt}$$

# Simulation


PL($t$)
PL$^{opt}$
losses [W]
60,000
55,000
50,000
0
5
10
15
20
25
30
iteration

# Conclusions

- Leaderless distributed decision models have pros and cons
- PROS: robustness to external changes, highly self-adaptiveness and so need of a limited initial configuration
- CONS: sub-optimal performance can be obtained
- Only a distributed modeling is needed
- Simplified approximated models need to be obtained
- Convergence and performance analysis can be done (distance to optimum)

# Questions?