Port Hamiltonian Modeling of Power Networks

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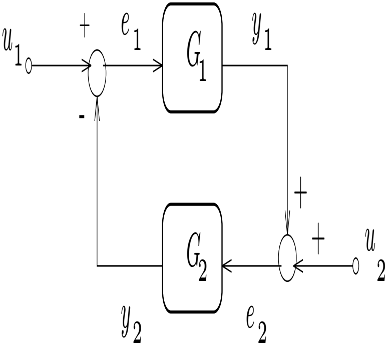
Outline

Control by interconnection

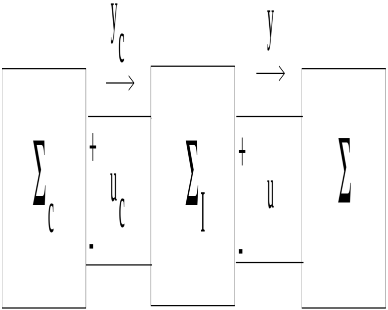
- Alternative to prevailing signal processing view–point of control
- Plant and controller viewed a energy processing dynamical systems
- Objectives:
  - exploit the interconnection to
  - shape energy and
  - modify dissipation
- Advantages:
  - handle on performance, not just stability
  - scalable for complex systems, e.g., smart grids
- Need for models that capture interconnection, energy and dissipation structures

Alternative paradigm for control

Control as signal processing

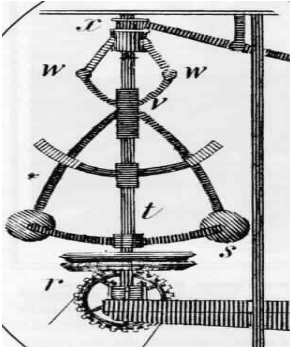


Control by interconnection

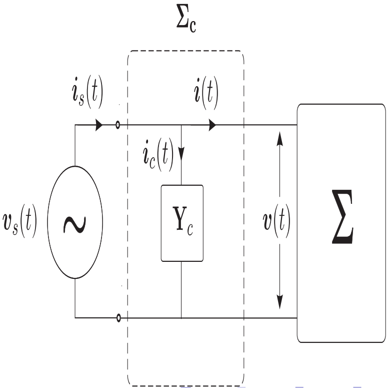


Classical examples

Old as control itself



Power factor compensation = energy equalization



Electrical energy network

- Most complex system built by mankind
  - Large scale
  - Nonlinear (highly stressed) dynamics
  - Deregulation brought:
    - Complex interactions
    - Non–negligible, switching loads
    - Harmonic distortion
- Recent blackouts: India 2012, Brazil 2009, Italy 2003 etc..

Power network consists of

1. Generators,
2. Loads,
3. Buses to which loads and generators are connected,
4. Transmission lines,
5. Switch-gear equipment . . .

Power network transient stability problem

- Ability to regain a state of operating equilibrium after being subjected to a physical disturbance, with system variables bounded $\Rightarrow$ Enlargement of domain of attraction
- Existing analysis and control techniques require simple models
- Modeling assumptions made in the classical literature
  - Linearity of the loads
  - Neglecting transmission losses
  - Neglecting transient phenomena on the network or the generators stator flux
  - Reduced models for generator dynamics: second order (Swing equation)) etc ...

Power network viewed as graph

Graph

- Edges: Generators, loads, transmission lines
- Nodes: Buses
- There is one reference bus (ground potential)
- Every generator and load edge ends at the reference bus, and
- the transmission line edges are in between the other generator and load buses
- The transmission lines define a reduced graph between the generator and load nodes.

Note: Edges are dynamic

Power network modeling

Example: Two generators and two loads, connected by five transmission lines. (N denotes reference bus.)

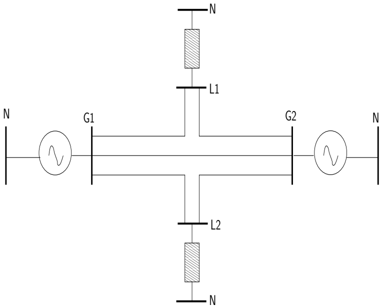


Figure: Example network

Power network

The corresponding graph: the blue part is the reduced graph.

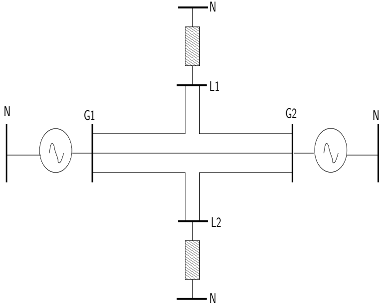


Figure: Example network

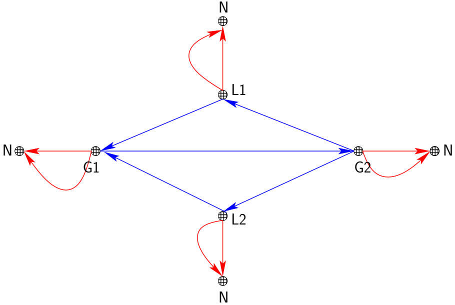


Figure: Graph of example network

Edge dynamics

Each edge element is represented as a

port-Hamiltonian system

$$\dot{x} = [\mathcal{J}(x) - \mathcal{R}(x)]\nabla H(x) + g(x)u,$$

$$y = g^T(x)\nabla H(x)$$

where x is the state, $\mathcal{J}^t(x) = -\mathcal{J}(x), \mathcal{R}^t(x) = \mathcal{R}(x) \geq 0$, and $H(x)$ are the interconnection, damping and energy functions, respectively.

The interconnection of all these port-Hamiltonian systems using Kirchhoff's laws will result in a total port-Hamiltonian system.

Port-Hamiltonian model of generator

- $\Psi_s = \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix}$-stator flux linkages
- $\Psi_r = \begin{bmatrix} \psi_f \\ \psi_{kd} \\ \psi_{kq} \end{bmatrix}$-rotor flux linkages
- p-momentum of rotor
- θ-rotor angle
- $H = \frac{1}{2}\begin{bmatrix} \Psi_s^t & \Psi_r^t \end{bmatrix} L^{-1}(\theta) \begin{bmatrix} \Psi_s^t & \Psi_r^t \end{bmatrix}^t + \frac{1}{2J}p^2$-total energy

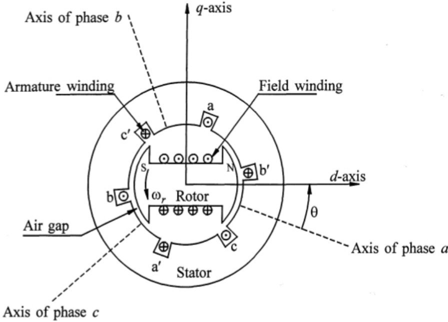


Figure: Generator

Port-Hamiltonian model of each Generator

$$\begin{bmatrix} \dot{\Psi}_{es} \\ \hline \dot{\Psi}_{er} \\ \hline \dot{p}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} -R_{es} & 0 & 0 & 0 \\ \hline 0 & -R_{er} & 0 & 0 \\ \hline 0 & 0 & -d_e & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \nabla_{(\Psi_{es},\Psi_{er},p_e,\theta_e)} H_e$$

$$+ \left[\begin{array}{c|c|c} \mathbb{I}_3 & 0_{3\times 1} & 0_{3\times 1} \\ \hline 0_{3\times 3} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & 0_{3\times 1} \\ \hline 0_{2\times 3} & 0_{2\times 1} & \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{array}\right] \begin{bmatrix} V_e \\ E_{ef} \\ T_{em} \end{bmatrix}$$

$$H_e = \frac{1}{2} \begin{bmatrix} \Psi_{es}^t & \Psi_{er}^t \end{bmatrix} L^{-1}(\theta_e) \begin{bmatrix} \Psi_{es}^t & \Psi_{er}^t \end{bmatrix}^t + \frac{1}{2J_e} p_e^2 .$$

Load models

In principle we can take many different load models.
For simplicity we will only consider an Ohmic load:

$$-I_L = R_L^{-1} V_L$$

Transmission line model

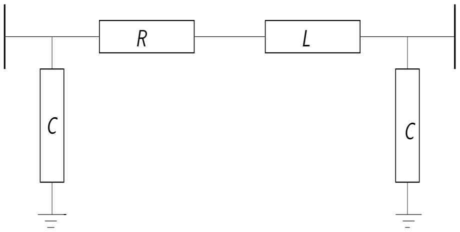


Figure: Π-model of transmission line

- $R - L$ series circuit

$$\begin{aligned} \dot{\Psi}_e &= -R_e \nabla_{\Psi_e} H_e(\Psi_e) + V_e \\ I_e &= \nabla_{\Psi_e} H_e(\Psi_e) \\ H_e(\Psi_e) &= \frac{1}{2} \Psi_e^T L_e^{-1} \Psi_e \end{aligned}$$

- Two capacitors

$$\begin{aligned} \dot{Q}_e &= I_e \\ V_e &= \nabla_{Q_e} H_e(Q_e) \\ H_e(Q_e) &= \frac{1}{2} Q_e^T C_e^{-1} Q_e. \end{aligned}$$

Interconnection laws

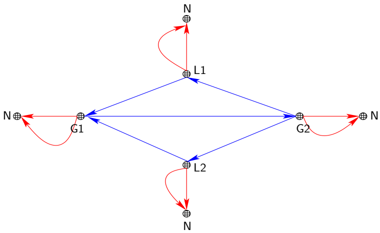


Figure: Graph of example network

- Incidence matrix of graph

$$M = \begin{bmatrix} \mathbb{I}_{3g} & 0_{3g\times 31} & M_1 & \mathbb{I}_{3g} & 0_{3g\times 31} \\ 0_{31\times 3g} & \mathbb{I}_{31} & M_2 & 0_{31\times 3g} & \mathbb{I}_{31} \\ -1^t_{3g} & -1^t_{31} & 0_{1\times 3T} & -1^t_{3g} & -1^t_{31} \end{bmatrix}$$

- Interconnection constraints (Kirchhoff's laws)

$$\begin{aligned} M\tilde{I} &= 0 \\ \tilde{V} &= M^T V \end{aligned}$$

V, $\tilde{V}$ node, edge voltages and $\tilde{I}$ are edge currents

- $\begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$ is incidence matrix capturing the reduced graph.

Elimination of constraints

1. To eliminate KVL:
   - Node voltages=voltages across capacitors at their nodes=gradient of the Hamiltonian with respective charges.
   - Voltages at stator terminal of generators=voltages across the generation side capacitors.
   - Voltages across transmission line $R - L$ circuit= $\begin{bmatrix} M_1^t & M_2^t \end{bmatrix} \begin{bmatrix} V_G \\ V_L \end{bmatrix}$
2. Similar substitutions to eliminate KCL

Complete model

After eliminating the interconnection constraints we have the complete model, with H the total energy:

$$
\begin{bmatrix} \dot{\Psi}_{SG} \\ \dot{\Psi}_T \\ \dot{Q}_{GT} \\ \dot{Q}_{LT} \\ \dot{\Psi}_{RG} \\ \dot{p}_G \\ \dot{\Theta}_G \end{bmatrix} = \begin{bmatrix} -R_{SG} & 0 & \mathbb{I} & 0 & 0 & 0 & 0 \\ 0 & -R_T & M_1^t & M_2^t & 0 & 0 & 0 \\ -\mathbb{I} & -M_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -M_2 & 0 & -R_L^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -R_{rG} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -D & -\mathbb{I} \\ 0 & 0 & 0 & 0 & 0 & \mathbb{I} & 0 \end{bmatrix} \nabla H + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ K & 0 \\ 0 & \mathbb{I} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_f \\ T_m \end{bmatrix}
$$

$$
\begin{bmatrix} I_f \\ \omega_G \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & K^t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbb{I} & 0 \end{bmatrix} \nabla H
$$

$$
H = \frac{1}{2} \begin{bmatrix} \Psi_{SG} \\ \Psi_{RG} \end{bmatrix}^t L(\Theta_G)^{-1} \begin{bmatrix} \Psi_{SG} \\ \Psi_{RG} \end{bmatrix} + \frac{1}{2} p_G^t J_G^{-1} p_G + \frac{1}{2} \Psi_T^t L_T^{-1} \Psi_T + \frac{1}{2} Q_{GT}^t C_G^{-1} Q_{GT} + \frac{1}{2} Q_{LT}^t C_L^{-1} Q_{LT}
$$

Complete model

In shorthand notation we have the port-Hamiltonian model

$$\dot{x} = [\mathcal{J} - \mathcal{R}]\nabla H(x) + gu$$
$$y = g^t \nabla H(x)$$

where

$$\mathcal{J} = \begin{bmatrix} 0 & 0 & \mathbb{I} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_1^t & M_2^t & 0 & 0 & 0 \\ -\mathbb{I} & -M_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -M_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mathbb{I} \\ 0 & 0 & 0 & 0 & 0 & \mathbb{I} & 0 \end{bmatrix}$$

Remarks

- Other port-Hamiltonian subsystems can be added like capacitor banks, transformers etc.
- Another model of the transmission line, e.g., partial differential equation models.
- Other load models.
- A different (simpler) port-Hamiltonian model of the generator.
- Techniques like Kron reduction can be used to simplify the graph.

Benefits of pH modelling

- It is a unified approach for modelling (electrical, mechanical etc. and their combination)
- Explicit use of the true energy of the system (useful for stability studies)
- Captures the structure and dissipation of the system ($\mathcal{J}$ and $\mathcal{R}$ matrices)
- Easily scalable

Current work

- Use of port-Hamiltonian model for stability analysis [3].
- Major problem: this concerns the stability of a forced system.
- Furthermore, there are no forced equilibria, but forced periodic motions.
- Using the model to study stability and fault diagnosis dependency on interconnection structure

[3] F. Shaik et al., On Port-Hamiltonian Modeling of the Synchronous Generator and Ultimate Boundedness of Its Solutions, to appear in *4th IFAC Workshop on Lagrangian and Hamiltonian Methods for Non Linear Control*

Thank you