On power sharing and stability in autonomous inverter-based microgrids

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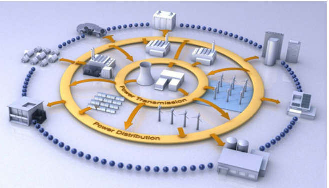
Motivation

- An increasing amount of renewable energy sources is present in the electrical grid
- Most of these sources are small-scale distributed generation units connected at the low (LV) and medium voltage (MV) levels via inverters
- Physical characteristics of such power electronic devices largely differ from characteristics of conventional electrical generators
- Moreover many of these generation units are intermittent by nature



Energy transmission and distribution

Source: siemens.com

$\leadsto$ Increasing use of renewable energy sources with intermittent generation requires more flexible ("smarter") ways of balancing power consumption and generation

The microgrid concept

Microgrids . . .

- are local power distribution networks
- include various renewable microsources and loads
- typically operate at low (LV) and medium voltage (MV) level
- from a “global” point of view, behave as a single controllable generator or load
- can operate in grid-connected or stand-alone mode
- have been identified as a key component in future electrical networks (Farhangi ’10)

Stability and power sharing in a microgrid

- In this talk we focus on the problem of guaranteeing *voltage and frequency stability* for an inverter-based microgrid under droop-like control by providing additional decentralized feedback
- *Power sharing*:
  Upon load changes in the system, how should the different generation units in the network adjust their output power in order to fulfill the demand while satisfying a desired power distribution
- *Requirement*:
  Achieve these objectives in a decentralized way without communication among units and allowing for a plug-and-play-like operation

Review: Conventional droop control

- A control solution widely used to tackle the problem of power sharing in large power systems is *droop control*
- Typical droop control in inverter-based systems is implemented according to

$$\omega = \omega_{\text{nom}} - k_P(P - P_{\text{nom}})$$
$$V = V_{\text{nom}} - k_Q(Q - Q_{\text{nom}}),$$

where ω is the inverter input frequency, V the inverter input voltage magnitude, ω_{nom} the nominal frequency, V_{nom} the nominal voltage magnitude, P, Q the active and reactive output power and $P_{\text{nom}}, Q_{\text{nom}}$ the nominal output powers

- The dynamic stability of microgrids under droop control is a main concern of many researchers (Filho *et. al.* '09)
- It has so far only been analyzed for linearized systems (e.g. Coelho *et. al.* '02) or special cases (parallel inverters connected to a single load in a lossless system with constant bus voltages (Simpson-Porco *et. al.* '12))

Preliminaries

- A suitable model of an inverter at node i can be given as

$$
\begin{aligned}
\dot{\delta}_i &= u_i^{\mathrm{a}} \\
\dot{V}_i &= \frac{1}{\tau_{V_i}}(-V_i + V_i^{\mathrm{d}} + u_i^{\mathrm{b}}) \\
\dot{\tilde{P}}_i &= \frac{1}{\tau_{P_i}}\left(-\tilde{P}_i + P_i\right) \\
\dot{\tilde{Q}}_i &= \frac{1}{\tau_{P_i}}\left(-\tilde{Q}_i + Q_i\right)
\end{aligned}
$$

δ_i	phase angle
V_i	voltage magnitude
V_i^{d}	desired (nominal) voltage magnitude
P_i	active power
Q_i	reactive power
$\tilde{P}_i$	measured active power
$\tilde{Q}_i$	measured reactive power
τ_{P_i}	time constant of meas. filter
τ_{V_i}	time constant of low pass filter to consider input delay in V_i ($\tau_{V_i} \ll \tau_{P_i}$)

- All phase angles are expressed with respect to an arbitrary rotating reference frame with angular velocity $\dot{\delta}_{\mathrm{nom}} = \omega_{\mathrm{nom}}$

Preliminaries ctd.

- Power exchange at node i

$$P_i = \sum_{j=1}^{n} \kappa_{ij}\alpha_{ij} V_i V_j |Y_{ij}| \cos(\delta_i - \delta_j - \phi_{ij})$$

$$Q_i = \sum_{j=1}^{n} \kappa_{ij}\alpha_{ij} V_i V_j |Y_{ij}| \sin(\delta_i - \delta_j - \phi_{ij})$$

$\lvert Y_{ij}\rvert$	magnitude of admittance between nodes i and j
$(\alpha_{ij} - 1)$	multiplicative uncertainty in $\lvert Y_{ij}\rvert$
ϕ_{ij}	admittance angle
V_i	voltage magnitude at node i
V_j	voltage magnitude at node j
n	number of nodes in network

$$\kappa_{ij} = \begin{cases} 1 & \text{node } i \text{ connected with node } j \\ 0 & \text{node } i \text{ not connected with node } j \end{cases}$$

- Since we are interested in the stability around an equilibrium point $y_i^e = (\delta_i^e, V_i^e, \tilde{P}_i^e, \tilde{Q}_i^e)$, $i = 1, \cdots, n$, we define our state variables as follows

$$\begin{aligned} x_{i_1} &= \delta_i - \delta_i^e, & x_{i_2} &= V_i - V_i^e, & \\ x_{i_3} &= \tilde{P}_i - \tilde{P}_i^e, & x_{i_4} &= \tilde{Q}_i - \tilde{Q}_i^e, & i &= 1, \ldots n. \end{aligned}$$

Preliminaries ctd.

- Equilibrium points are usually not known explicitly in power systems $\rightsquigarrow$ no state-feedback possible
- Moreover in power sharing, a change of equilibrium point should typically be achieved with respect to desired operating values $y_i^{\mathrm{d}} = (\delta_i^{\mathrm{d}}, V_i^{\mathrm{d}}, \tilde{P}_i^{\mathrm{d}}, \tilde{Q}_i^{\mathrm{d}})$ e.g., the nominal operating values

- For that matter, we define the deviations of the system variables with respect to their desired values as

$$z_{i_1} = \delta_i - \delta_i^{\mathrm{d}}, \qquad z_{i_2} = V_i - V_i^{\mathrm{d}},$$
$$z_{i_3} = \tilde{P}_i - \tilde{P}_i^{\mathrm{d}}, \qquad z_{i_4} = \tilde{Q}_i - \tilde{Q}_i^{\mathrm{d}}, \qquad i = 1, \dots n$$

- The choice of the desired variables does *not* affect stability of the system and can thus be made arbitrarily, but will affect the steady-state after a change in load

Nonlinear model of microgrids

Error dynamics for node i

$$\dot{x}_i(t) = A_i x_i(t) + B_i \Delta u_i(t) + \sum_{j=1}^{n} \kappa_{ij} G_i f_{ij}(x_i, x_j), \quad \Delta u_i(t) = \begin{pmatrix} u_i^{\mathrm{a}} - u_i^{\mathrm{a,e}} \\ u_i^{\mathrm{b}} - u_i^{\mathrm{b,e}} \end{pmatrix} \quad (1)$$

$$\begin{aligned} u_i(t) &= \begin{pmatrix} u_i^{\mathrm{a}} & u_i^{\mathrm{b}} \end{pmatrix}^T = u_{i_{\mathrm{droop}}}(t) + u_{i_{\mathrm{stab}}}(t) \\ &= -K_{i_1} z_i(t) - K_{i_2} z_i(t) = -\begin{pmatrix} 0 & K_{PQ_i} \end{pmatrix} z_i - \begin{pmatrix} K_{i_{\mathrm{stab}}} & 0 \end{pmatrix} z_i \end{aligned}$$

- Different structures can be considered for the matrix K_{PQ_i}
- Our formal approach is valid for any structure of K_{PQ_i}
- In the following we focus for simplicity on the diagonal matrix:

$$K_{PQ_i} = \begin{pmatrix} k_{P_i} & 0 \\ 0 & k_{Q_i} \end{pmatrix}$$

- Under this droop-control gain matrix, the phase angle (x_{i_1}) of the inverter is modified according to the active power (z_{i_3}), while the reactive power (z_{i_4}) modifies the voltage magnitude (x_{i_2})

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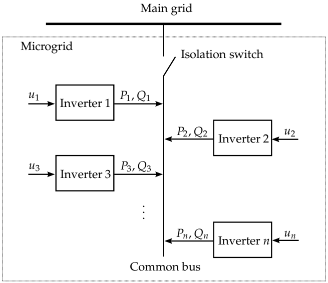
Problem statement

Problem definition:
Given a desired droop-like control law $u_{i_{\text{droop}}} = -K_{i_1} z_i$ and a set of desired droop-gain matrices

$$K_{i_1} \in \Gamma_i : \left\{ [0, \text{diag}(k_{P_i}, k_{Q_i})] \mid 0 \leq k_{P_i} \leq \bar{k}_{P_i}, 0 \leq k_{Q_i} \leq \bar{k}_{Q_i} \right\},$$

design a decentralized control $u_{i_{\text{stab}}} = -K_{i_2} z_i$ that stabilizes the system (1) (for $i = 1, \ldots, n$).



Decentralized linear control and power sharing

- The nonlinear multi-inverter system (1) is stabilizable via decentralized linear feedback control for any W_i, M_i, N_i, $i = 1, \ldots, n$ satisfying the following conditions $\forall K_{i_1} \in \Gamma_i$:

$$\begin{bmatrix} \hat{A}_i & W_i \bar{K}_{i_1}^T & B_i & G_i & W_i F_i^T \\ \bar{K}_{i_1} W_i & -\frac{1}{\epsilon} I & 0 & 0 & 0 \\ B_i^T & 0 & -\epsilon I & 0 & 0 \\ G_i^T & 0 & 0 & -\tilde{\rho} I & 0 \\ F_i W_i & 0 & 0 & 0 & -\rho I \end{bmatrix} < 0, \qquad \begin{array}{c} M_i C_i = C_i W_i, \\ W_i > 0, \end{array}$$

  where $\hat{A}_i = W_i A_i^T + A_i W_i - B_i N_i C_i - C_i^T N_i^T B_i^T$, $\bar{K}_{i1} = \max_{K_{i1}}(\Gamma_i)$, $C_i = \begin{bmatrix} I & 0 \end{bmatrix}$ and $\tilde{\rho} = (\rho \sum_{j=1}^n \kappa_{ij})^{-1}$

- Moreover a stabilizing control law is then given by:
  $u_{i_{\text{stab}}} = -K_{i_2} z_i = -\begin{pmatrix} N_i M_i^{-1} & 0 \end{pmatrix} z_i$

- It can further be shown that system (1) is always stabilizable via diagonal static output feedback:

$$u_{i_{\text{stab}}} = -K_{i_2} z_i = -\begin{pmatrix} K_{i_{\text{stab}}} & 0 \end{pmatrix} z_i = -\begin{pmatrix} k_{\delta_i} & 0 & 0 & 0 \\ 0 & k_{V_i} & 0 & 0 \end{pmatrix} z_i$$

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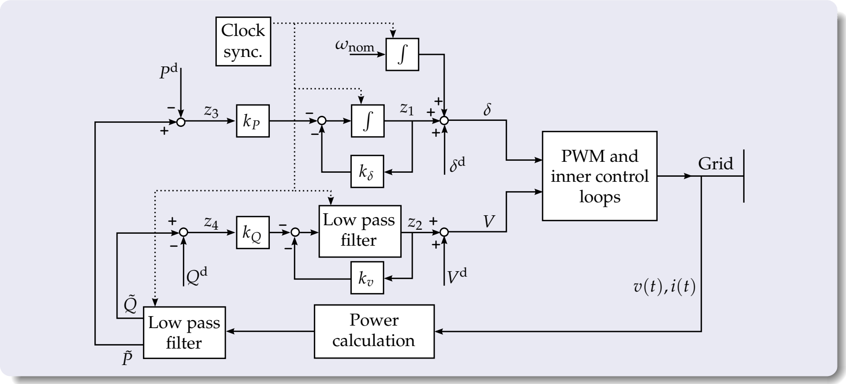
Sketch of proof

- The proof draws inspiration from Jiang *et.al* ’97, Wang *et.al* ’98 and Zecevic *et.al* ’04
- Derive quadratic bounds for nonlinearities
- Define block diagonal Lyapunov function: $V = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} x_i^T \Phi_i x_i$
- Defining $W_i = \Phi_i^{-1}$, $W_i > 0$, using the Schur complement and pre- and postmultiplying with W_i (Siljak *et.al.* ’00) the condition on $\dot{V} < 0$ can be rewritten as n BMIs
- Applying the W-Problem (Crusius *et.al.* ’99) leads to the proposed LMI optimization problem
- The specific structure of $K_{i_{\text{stab}}}$ resembles the case of decentralized static output feedback control design

Proposed control structure



Decentralized control structure with clock synchronization signal for referencing

Remarks

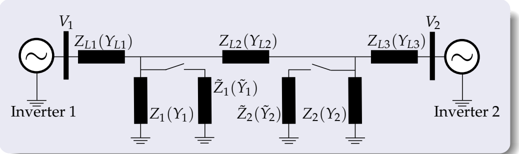
- Stabilizing gains k_{δ_i}, k_{V_i} restrict magnitude of user-selected droop gains k_{P_i}, k_{Q_i} in order to guarantee stability
- While our design method does guarantee overall network stability, we can not make any claims regarding the power sharing performance (the latter is part of our on-going investigations)

- The approach guarantees zero steady-state frequency deviation, which removes the need for secondary control in charge of frequency restoration

- Formally, only an upper bound of the admittance matrix of the underlying network needs to be known (as our approach also accounts for uncertainties in the interconnections)

Test system setup

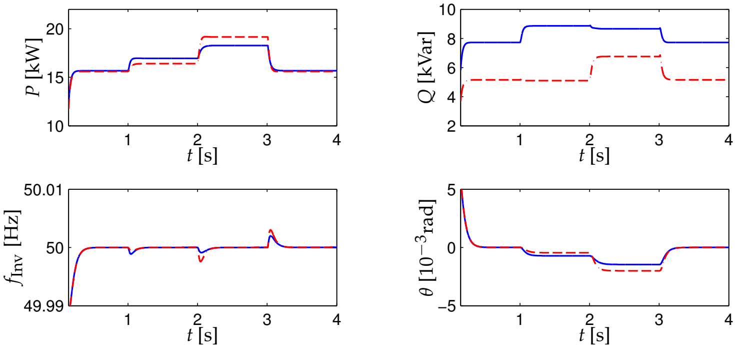
- Test system: two inverters having each a local load represented by a frequency dependent impedance and being connected via an LV line
- Design parameters: $k_{P_i} \in [0, 10^{-2}]$, $k_{Q_i} \in [0, 1]$, $V_{\max} = 1.2 V_{\text{nom}}$, $\alpha_{ij} = 1.1$, $\{i, j\} \in \{1, 2\}$



	Inverter 1	Inverter 2
Voltage magnitude and phase angle	$V_1^d = 232$ V $\delta_1^d = 0$ rad	$V_2^d = 228.5$ V $\delta_2^d = -10^{-4}$ rad
Active and reactive power	$\tilde{P}_1^d = 15.68$ kW $\tilde{Q}_1^d = 7.73$ kVar	$\tilde{P}_2^d = 15.6$ kW $\tilde{Q}_2^d = 5.16$ kVar
Time constant low pass filter	$\tau_{P_1} = 0.0265$ s $\tau_{V_1} = 10^{-3}$ s	$\tau_{P_2} = 0.0265$ s $\tau_{V_2} = 10^{-3}$ s
Control gains via LMI approach	$k_{\delta_1} = 17.72\ \frac{1}{\text{s}}$ $k_{V_1} = 6.98$ $k_{P_1} = 10^{-2}\ \frac{\text{rad}}{\text{skW}}$ $k_{Q_1} = 1\ \frac{\text{V}}{\text{kVar}}$	$k_{\delta_2} = 17.72\ \frac{1}{\text{s}}$ $k_{V_2} = 7.05$ $k_{P_2} = 10^{-2}\ \frac{\text{rad}}{\text{skW}}$ $k_{Q_2} = 1\ \frac{\text{V}}{\text{kVar}}$

Load impedances $Z_i = R_i + j\omega L_i$ $Z_1 = (4 + j1.95)\ \Omega$ $Z_2 = (2.24 + j0.79)\ \Omega$ $\tilde{Z}_1 = (20 + j9.7)\ \Omega$ $\tilde{Z}_2 = (11.2 + j3.7)\ \Omega$
Line impedances $Z_{Li} = R_i + j\omega L_i$ $Z_{L1} = (0.01 + j0.05)\ \Omega$ $Z_{L3} = (0.01 + j0.04)\ \Omega$ $Z_{L2} = (0.12 + j0.03)\ \Omega$
Nominal voltage $V_{\text{nom}} = 230$ V
Nominal frequency $f_{\text{nom}} = 50$ Hz

Simulation example



Test system with proposed control, Inverter 1 '–', Inverter 2 '- -'

- At $t = 1s$, $t = 2s$ and $t = 3s$ both inverters react to load changes by adjusting their power outputs
- Active power sharing is improved by over a factor $3/2$ with respect to the case without control
- System stabilizes with zero steady-state frequency deviation

Conclusions

- A decentralized feedback control design addressing the problem of voltage and frequency stability for a nonlinear inverter-based microgrid under droop-like control by providing additional decentralized feedback has been presented
- The control synthesis (also decentralized) is formulated as an LMI
- The design allows for a user-specified range for power sharing gains as well as line and load uncertainties
- Opposed to standard droop control, the approach guarantees zero steady-state frequency deviation and stability

For further details, please see:
J. Schiffer, A. Anta, T. D. Trung, J. Raisch, T. Sezi, *On power sharing and stability in autonomous inverter-based microgrids*, Proc. 51st IEEE Conf. on Decision and Control (CDC) (to appear), HI, USA

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