ROBUST TRANSIENT SYNCHRONIZATION OF POWER NETWORKS

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European Union area: 3236, 2 km²
Russian Federation area: 17 098 246 km²
More than 600 power stations with power more than 5 MW
More than 10200 transmission lines of 110 - 1150 kW
Total power generation capacity 218 235.8 MW in 2011
Total generated energy: about one trillion kilowatt hours of electricity per year
Emergence of ‘Smart Grid’

Development of ‘smart grid’ area is caused by the following factors:

- technological progress
- growth of customer requirements
- decreasing reliability of electrical networks
- increasing demands for energy efficiency and ecological safety
- etc.


Synchronization

Synchronization: - coincidence of the generator rotor speeds,
- reduction to zero the difference between active electrical power and mechanical input power of each generator

\[
\begin{bmatrix}
\dot{\delta}_i \\
\dot{\omega}_i \\
\Delta P_{ei} \\
\vdots
\end{bmatrix} = f\left(\delta_1, \ldots, \delta_k, \omega_1, \ldots, \omega_k, \Delta P_{e1}, \ldots, \Delta P_{ek}, \ldots\right), \quad i = 1, \ldots, k
\]

\(\delta_i\) - deflection of the rotor angle of the \(i\)th generator from synchronous mode

\(\omega_i\) - deflection of the speed of the \(i\)th generator from synchronous mode

\(\Delta P_{ei} = P_{ei} - P_{mi}\)

\(P_{ei}\) - the active electrical power delivered by the \(i\)th generator

\(P_{mi}\) - the mechanical input power of the \(i\)th generator

**Transient stability**

\[
\lim_{t \to \infty} \delta_i = const \quad \lim_{t \to \infty} \omega_i = 0 \quad \lim_{t \to \infty} \Delta P_{ei} = 0 \quad \left(\lim_{t \to \infty} (\omega_i - \omega_j) = 0\right)
\]

Existing results

Robust decentralized control

- Network models: sets of nonlinear 3rd order differential-algebraic equations (DAE).
- Measurables: rotor angles and speeds, active electrical power and mechanical power of generators.
- Class of faults: short-term changes in the resistance of the transmission line (short circuits faults, etc).
- Control approach: feedback linearization

- The results are similar to the results of Hill et. al.
- Network generator models are described by linear DAE of the third order.
Existing results


Transient stabilization of the power systems

- All network parameters are known.
- Network model: 3rd order DAE model for the generators; load model; the equations of transmission lines; equations of infinite buses.
- Measurables: the power angles, the relative speeds, the transient electromotive force (EMF) of generators.
- Control approach: energy shaping / interconnection and damping assignment (IDA-PBC)


Synchronization of electrical generators network

- Models of the generators are described by 2nd order differential equations.
- The power angles and the relative speeds of each generators are available to measurement.
- Control approach: passivity and speed gradient.
I. Speed-gradient-energy approach

Power network model

\[
\begin{aligned}
\dot{\delta}_i &= \omega_i, \\
\dot{\omega}_i &= -D_i \omega_i + P_{mi} - G_i E_i^2 - \sum_{j=1, j \neq i}^{N} \left( \alpha_{ij} \cos(\delta_i - \delta_j) + \beta_{ij} \sin(\delta_i - \delta_j) \right), \\
\dot{E}_i &= f_i + v_i.
\end{aligned}
\]  

(1)

where \(i=1,..N\), \(\alpha_{ij} = E_i E_j G_{ij}\), \(\beta_{ij} = E_i E_j B_{ij}\), \(i \neq j\).

\(\delta_i\) is the rotor angle,

\(\omega_i = \omega_0 - \omega_{Ri}\) ; \(\omega_{Ri}, \omega_0\) are the rotor speed and the synchronous speed,

\(E_i\) is the internal voltage in the quadrature axis of the \(i\)th generator,

\(v_i\) is the control signal (the field excitation signal),

\(f_i = F_i(\delta_1, \ldots, \delta_N; E_1, \ldots, E_N)\) – the known function,

\(D_i, P_{mi}, G_{ii}, G_{ij}, B_{ij}\) – the constant parameters.


Invariant

Let us neglect damping and cancel control:
\[ D_i = 0; \ G_{ij} = 0; \ B_{ij} = B_{ji}; \]  
\[ E_i = E_{di} = \sqrt{\frac{P_{mi}}{G_{ii}}}, \ (v_i = -f_i). \]

for all \( i,j=1,..,N \). Then the following function is an invariant of (1):
\[
H(\delta, \omega) = \frac{1}{2} \omega^T \omega + \sum_{i=1}^{N} \left( \sum_{j=i+1}^{N} \left( \beta_{ij} \left(1 - \cos(\delta_i - \delta_j)\right) + \alpha_{ij} \left(1 + \sin(\delta_i - \delta_j)\right) \right) \right),
\]
where \( \omega = (\omega_1, \omega_2, \ldots, \omega_N)^T \) and \( \delta = (\delta_1, \delta_2, \ldots, \delta_N)^T \).

Control goal: to achieve the desired level of (3) and approximate transient stability of (1):
\[
H \xrightarrow{t \to \infty} H_d, \ E_i \xrightarrow{t \to \infty} E_{di}, \]
\[ \delta_i \in \left(0; \frac{\pi}{2}\right), \ i = 1,..,N. \]
Control Algorithm

Introduce a new control

\[ u_i = v_i - f_i, \quad i = 1, \ldots, N, \]  

(5)

and a goal function

\[ Q = \frac{1}{2} \kappa (H - H_d)^2 + \sum_{i=1}^{N} \frac{1}{2} (E_i - E_{di})^2. \]  

(6)

Design control according to the speed-gradient algorithm:

\[ u_i = -\gamma_i \nabla u_i \dot{Q}, \quad i = 1, \ldots, N. \]  

(7)

Finally the control algorithm is

\[ v_i = -\gamma_i \kappa (Q - Q_d) \sum_{j=1,j\neq i}^{N} E_j B_{ij} (1 - \cos(\delta_i - \delta_j)) - \gamma_i (E_i - E_{di}) + \eta_i \sum_{j=1}^{N} \omega_j - f_i, \quad i = 1, \ldots, N. \]
Simulation results

The system (1) with $N=5$ and the following parameters is considered

$$D = [0; 0; 0; 0; 0]^T; \quad P = [6; 5; 5.5; 5.3; 5.8]^T; \quad G = \{G_{ii}\}_{i=1}^N = [3; 2; 2.5; 2.3; 2.8]^T;$$

$$G_{ij} = 0; \quad B_{ij} = 4; \quad Q_d = 0.6; \quad \gamma_i = 2; \quad \eta_i = 2; \quad \kappa = 6.$$

Initial conditions are as follows:

$$\delta = (\pi / 20; \pi / 10; \pi / 15; \pi / 18; \pi / 12)^T; \quad \omega = (0; 0; 0; 0; 0)^T;$$

$$E = (E_{d1} - 0.5; E_{d2}; E_{d3}; E_{d4}; E_{d5})^T = (0.9142; 1.5811; 1.4832; 1.5180; 1.4392)^T.$$
Simulation results
II. Auxiliary loop approach
DAE power network model[*]

Mechanical equations $i=1,...,k$

$$\dot{\delta}_i(t) = \omega_i(t), \quad \dot{\omega}_i(t) = -\frac{D_i}{2H_i} \omega_i(t) - \frac{\omega_0}{2H_i} \Delta P_{e_i}(t)$$  \hspace{1cm} (1)

Generator electrical dynamics $i=1,...,k$

$$\dot{E}_{qi}'(t) = \frac{1}{T_{d0i}'} \left( E_{fi}(t) - E_{qi}(t) \right)$$  \hspace{1cm} (2)

Electrical equations $i=1,...,k$

$$E_{fi}(t) = k_{ci} u_{fi}(t), \quad I_{di}(t) = -\sum_{j \in N_i} E_{qj}'(t) M_{ij} \cos(\delta_i(t) - \delta_j(t))$$

$$Q_{ei}(t) = -\sum_{j \in N_i} E_{qi}'(t) E_{qj}'(t) M_{ij} \cos(\delta_i(t) - \delta_j(t))$$

$$P_{e_i}(t) = \sum_{j \in N_i} E_{qi}'(t) E_{qj}'(t) M_{ij} \sin(\delta_i(t) - \delta_j(t))$$

$$E_{qi}(t) = x_{adi} I_{fi}(t) = E_{qi}'(t) - (x_{di} - x_{didi}) I_{di}(t)$$

$$I_{qi}(t) = \sum_{j \in N_i} E_{qj}'(t) M_{ij} \sin(\delta_i(t) - \delta_j(t))$$

$$V_{ii}(t) = \frac{1}{x_{dsi}} \sqrt{\left(E_{qi}'(t) - x_{didi} I_{di}(t)\right)^2 + \left(x_{didi} I_{qi}(t)\right)^2}$$

Assumptions
1. $\delta_i \in (0; \pi)$
2. $\varsigma \in \mathbb{Z}$, where $\varsigma$ is the vector of the unknown parameters of the equations (1)-(3);
   $\mathbb{Z}$ is known bounded set.
3. The quadratic axis currents signs $I_{qi}(t), i = 1, ..., k$ are known.
4. Only the rotor speed deflections $\omega_i(t), i = 1, ..., k$ are measured.

Controller design

The goal

\[
\lim_{t \to T} \delta_i(t) = \text{const} \quad \left| \omega_i(t) \right| < \varepsilon_1 \quad \left| \Delta P_{ei}(t) \right| < \varepsilon_2 \quad \left| \omega_i(t) - \omega_j(t) \right| < \varepsilon_3 \quad \left| \delta_i(t) - \delta_j(t) \right| < \pi
\]  

(4)

for \( t > T \), where \( \varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_3 > 0 \) are small numbers

Control system

Auxiliary loop

\[ Q_m(p)\bar{e}_i(t) = \chi u_{fi}(t), \quad i = 1, \ldots, k \]  

(5)

Observer [**]

\[ \dot{\xi}_i(t) = G_0\xi_i(t) + D_0(\bar{z}_i(t) - \zeta_i(t)), \quad \bar{z}_i(t) = L\xi_i(t), \quad i = 1, \ldots, k \]  

(6)

Control law

\[ u_{fi}(t) = -\chi^{-1}Q_m(p)\bar{e}_i(t), \quad i = 1, \ldots, k \]  

(7)

\[ e_i(t) = \sum_{j \in N_i} \left( \omega_i(t) - \omega_j(t) \right) \]

Main theoretical result

For any $\chi > 0$ there is a number $\mu_0 > 0$ such that the control system (5)-(7) ensures the goal (4) for the power network (1)-(3) for $\mu < \mu_0$.

## Simulation results

### Network parameters and control system


\[
\begin{align*}
4 \leq H_i &\leq 5.5 \\
6 \leq T'_{d0i} &\leq 8 \\
0.2 \leq x'_{di} &\leq 0.4 \\
3 \leq D_i &\leq 5 \\
1 \leq k_{ci} &\leq 3 \\
1.8 \leq x_{di} &\leq 2.4 \\
0.3 \leq M_{ij} &\leq 3 \\
-3 \leq E_{fi}(t) &\leq 6 \\
i =1, 2, 3, 4
\end{align*}
\]

### Control system

**Auxiliary loop**

\[
\left(p^2 + 4p + 4\right)\bar{\xi}_i(t) = -u_{fi}(t), \quad i = 1, 2, 3, 4
\]

**Observer**

\[
\begin{align*}
\dot{\xi}_{1i}(t) &= -\xi_{2i}(t) - 4 \cdot 100 (\xi_{1i}(t) - \zeta_i(t)), \\
\dot{\xi}_{2i}(t) &= -4 \cdot 100^2 (\xi_{1i}(t) - \zeta_i(t)), \\
i &= 1, 2, 3, 4
\end{align*}
\]

**Control law**

\[
u_{fi}(t) = \dot{\xi}_{2i}(t) + 4 \xi_{2i}(t) + 4 \xi_{1i}(t), \quad i = 1, 2, 3, 4
\]

**Error**

\[
e_i(t) = \sum_{j \in N_i} \left(\omega_i(t) - \omega_j(t)\right)
\]
Network parameters

**Alarm 1[*]**) before \( t = 1 \) s.,
\[ M_{12} = M_{21} = 0.4853 \text{ p.u.}, \]
when \( t = 1 \) s., \( M_{12} = M_{21} = 3 \) p.u.,
when \( t = 1.5 \) s.
\[ M_{12} = M_{21} = 0.4853 \text{ p.u.}; \]

**Alarm 2** before \( t = 10 \) s.
\[ M_{24} = M_{42} = 0.95 \text{ p.u.}, \]
when \( t = 10 \) s. fault in line 2-4

\[
\omega_0 = 314.159 \text{ rad/s}, \quad \omega_i(0) = 0 \text{ rad/s}, \quad \Delta P_{ei}(0) = 0 \text{ p.u.} \]

<table>
<thead>
<tr>
<th>Generator</th>
<th>( D_i ), p.u.</th>
<th>( H_i ), s.</th>
<th>( T_{d0i}' ), s.</th>
<th>( x_{di} ), p.u.</th>
<th>( x_{di}' ), p.u.</th>
<th>( P_{m0i} ), p.u.</th>
<th>( V_{t0i} ), p.u.</th>
<th>( k_{ci} ), p.u.</th>
<th>( \delta_i(0) ), rad</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_1 )</td>
<td>5</td>
<td>4</td>
<td>1,7</td>
<td>1,863</td>
<td>0,257</td>
<td>0,9</td>
<td>1</td>
<td>1</td>
<td>( \pi / 3 )</td>
</tr>
<tr>
<td>( G_2 )</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>2,17</td>
<td>0,32</td>
<td>0,8</td>
<td>0,9</td>
<td>1</td>
<td>( 11\pi / 36 )</td>
</tr>
<tr>
<td>( G_3 )</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>2,01</td>
<td>0,28</td>
<td>1</td>
<td>1,1</td>
<td>1</td>
<td>( 13\pi / 3 )</td>
</tr>
<tr>
<td>( G_4 )</td>
<td>4,5</td>
<td>5,2</td>
<td>2,1</td>
<td>2,07</td>
<td>0,35</td>
<td>0,85</td>
<td>0,87</td>
<td>1</td>
<td>( 7\pi / 20 )</td>
</tr>
</tbody>
</table>

Simulation results

\[ \delta_i(t), \quad \text{deg} \]

\[ w_i(t), \quad \text{rad/s} \]

\[ V_{ti}(t), \quad \text{p. u.} \]
